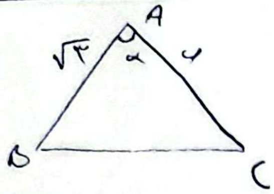
باران کاظمی — تکلیف شماره ۲۷

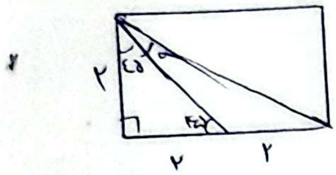
۱)



$\frac{1}{2}\times\sqrt{3}\times 6\times\sin\alpha = 4.5 \longrightarrow \sin\alpha = \frac{9^3}{2\times 6\sqrt{3}} = \frac{3\sqrt{3}}{2\times 3} = \frac{\sqrt{3}}{2}$

$\sin\alpha = \frac{\sqrt{3}}{2}$ → $\alpha = 60$ یا $\alpha = 120 \longrightarrow \frac{120}{60} = \boxed{2}$

۲)



$\cos(45+\alpha) = \cos45\cos\alpha - \sin45\sin\alpha = \frac{2}{2\sqrt{5}}$

$\frac{\sqrt{2}}{2}(\cos\alpha - \sin\alpha) = \frac{2}{2\sqrt{5}} \longrightarrow \cos\alpha - \sin\alpha = \frac{2\sqrt{5}}{5\sqrt{2}}$ ①

$\sin(45+\alpha) = \sin45\cos\alpha + \sin\alpha\cos45 = \frac{\sqrt{2}}{2}(\cos\alpha + \sin\alpha) = \frac{4}{2\sqrt{5}}$

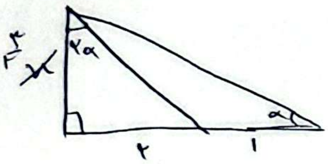
$\longrightarrow \cos\alpha + \sin\alpha = \frac{4\sqrt{5}}{5\sqrt{2}}$ ②

①،② →
$\cos\alpha - \sin\alpha = \frac{2\sqrt{5}}{5\sqrt{2}}$
$\cos\alpha + \sin\alpha = \frac{4\sqrt{5}}{5\sqrt{2}}$
→ $\sin\alpha = \frac{\sqrt{5}}{5\sqrt{2}}$

$2\cos\alpha = \frac{6\sqrt{5}}{5\sqrt{2}} \longrightarrow \cos\alpha = \frac{3\sqrt{5}}{5\sqrt{2}}$

$\boxed{\cot\alpha = 3}$

۳)



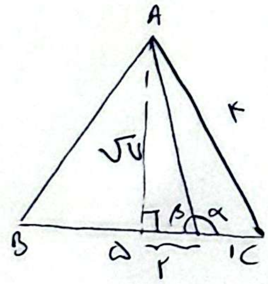
$\tan 2\alpha = \frac{2}{x} \qquad \tan\alpha = \frac{x}{3}$

$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} \longrightarrow \frac{2}{x} = \frac{\frac{2x}{3}}{1-\frac{x^2}{9}} \Rightarrow \frac{2}{x} = \frac{\frac{2x}{3}}{\frac{9-x^2}{9}}$

$\longrightarrow \frac{2}{x} = \frac{6x}{9-x^2} \rightarrow 6x^2 = 18 - 2x^2 \rightarrow 8x^2 = 18 \rightarrow x^2 = \frac{9}{4} \rightarrow x = \frac{3}{2}$

$\tan\alpha = \frac{\frac{3}{2}}{3} = \boxed{\frac{1}{2}}$

۴)



$\tan\alpha = \tan(180-\beta) = -\tan\beta = \boxed{-\frac{\sqrt{7}}{2}}$

۵)

$2\sin^2x + \cos^2x = \frac{4}{3}$
$\sin^2x + \cos^2x = 1 \longrightarrow \cos^2x = \frac{2}{3}$
$\sin^2x = \frac{1}{3}$

$\tan^2x = \frac{\sin^2x}{\cos^2x} = \frac{\frac{1}{3}}{\frac{2}{3}} = \boxed{2}$

(۶

$$\frac{\sin^4\alpha+4\cos^2\alpha}{1+\cos^2\alpha}-\frac{\cos^4\alpha+4\sin^2\alpha}{1+\sin^2\alpha}=\frac{\sin^4\alpha+4(1-\sin^2\alpha)}{1+1-\sin^2\alpha}-\frac{\cos^4\alpha+4(1-\cos^2\alpha)}{1+1-\cos^2\alpha}$$

$$=\frac{(\sin^4\alpha-4\sin^2\alpha+4)}{2-\sin^2\alpha}-\frac{\cos^4\alpha-4\cos^2\alpha+4}{2-\cos^2\alpha}=\frac{(\sin^2\alpha-2)^2}{-(\sin^2\alpha-2)}-\frac{(\cos^2\alpha-2)^2}{-(\cos^2\alpha-2)}$$

$$=2-\sin^2\alpha-2+\cos^2\alpha=\cos^2\alpha-\sin^2\alpha=\boxed{\cos2\alpha}$$

(۷

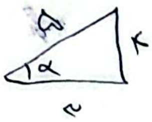
$$\sin(\frac{9\pi}{2}+\alpha)\cos(\frac{7\pi}{2}-\alpha)-\tan(\alpha-\frac{3\pi}{2})$$

$$=\sin(\frac{9\pi}{2}+\alpha)\cos(\frac{7\pi}{2}-\alpha)+\tan(\frac{3\pi}{2}-\alpha)$$

$$=(\cos\alpha)(-\sin\alpha)+(\cot\alpha)$$

$$=(-\frac{3}{5})(-)(-\frac{4}{5})+\frac{3}{4}$$

$$=-\frac{12}{25}+\frac{3}{4}=\frac{-48+75}{100}=\boxed{0/27}$$



(۸

$$3\cos4x+\sqrt2\sin x-\sqrt2\cos x=3\cos\frac{\pi}{3}+\sqrt2\sin(\frac{\pi}{4}-\frac{\pi}{6})-\sqrt2\cos(\frac{\pi}{4}-\frac{\pi}{6})$$

$$=3(\frac{1}{2})+\underbrace{\sqrt2(\frac{\sqrt6-\sqrt2}{4})}_{\frac{\sqrt{12}}{4}-\frac{1}{2}}-\underbrace{\sqrt2(\frac{\sqrt6+\sqrt2}{4})}_{\frac{\sqrt{12}}{4}-\frac{1}{2}}=\frac{3}{2}-\frac{2}{2}=\boxed{\frac{1}{2}}$$

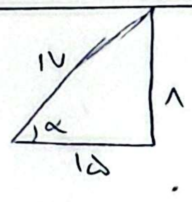
$$\sin(\frac{\pi}{4}-\frac{\pi}{6})=\sin\frac{\pi}{4}\cos\frac{\pi}{6}-\sin\frac{\pi}{6}\cos\frac{\pi}{4}=(\frac{\sqrt2}{2})(\frac{\sqrt3}{2})-(\frac{1}{2})(\frac{\sqrt2}{2})=\frac{\sqrt6-\sqrt2}{4}$$

$$\cos(\frac{\pi}{4}-\frac{\pi}{6})=\cos\frac{\pi}{4}\cos\frac{\pi}{6}+\sin\frac{\pi}{4}\sin\frac{\pi}{6}=(\frac{\sqrt2}{2})(\frac{\sqrt3}{2})+(\frac{\sqrt2}{2})(\frac{1}{2})=\frac{\sqrt6+\sqrt2}{4}$$

(۹

$$\tan(\frac{\alpha}{2})=\frac{1}{4}$$

$$\tan\alpha=\frac{2\tan(\frac{\alpha}{2})}{1-\tan^2(\frac{\alpha}{2})}=\frac{\frac{1}{2}}{\frac{15}{16}}=\frac{8}{15}$$



$$\frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}}=\frac{\frac{136-120}{255}}{-\frac{7}{17}}=\frac{\frac{16}{15}}{-\frac{7}{1}}=\boxed{-\frac{16}{105}}$$

(۱۰

$$\frac{\cos\alpha}{\sin2\alpha}>0 \qquad \underline{\cos\alpha>0}$$

$$2\sin\alpha<\sin2\alpha\to 0<2\sin\alpha\underbrace{(\cos\alpha-1)}_{\le0}$$

$$\downarrow$$

$$\underline{\sin\alpha<0}$$

[illegible]