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نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۲۷ کلاس یازدهم .....

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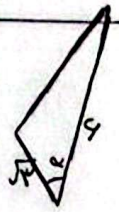

$S = \frac{1}{2}ab\sin\alpha \rightarrow 4\sqrt{3} = \frac{1}{2}\times 6\times\sqrt{4}\times\sin\alpha \rightarrow \sin\alpha = \frac{\sqrt{3}}{2}$

$\Rightarrow \alpha_1 = \frac{2\pi}{3}$ و $\alpha_2 = \frac{\pi}{3}$

$\frac{\alpha_{max}}{\alpha_{min}} = \frac{\frac{2\pi}{3}}{\frac{\pi}{3}} = 2$ ✓

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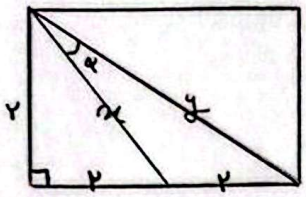

$x^2 = 4+4 \rightarrow x^2 = 8 \Rightarrow x = 2\sqrt{2}$

$y^2 = 4+16 = 20 \rightarrow y = 2\sqrt{5}$

$c = \sqrt{a^2+b^2-2ab\cos\alpha} \rightarrow 2 = \sqrt{8+20-8\sqrt{10}\cos\alpha}$

$\Rightarrow 4 = 28 - 8\sqrt{10}\cos\alpha \rightarrow \cos\alpha = \frac{3}{\sqrt{10}} \rightarrow \sin\alpha = \frac{1}{\sqrt{10}}$

$\cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{\frac{3}{\sqrt{10}}}{\frac{1}{\sqrt{10}}} = 3$ ✓

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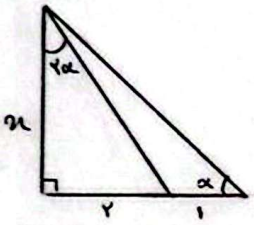

$\cot 2\alpha = \frac{x}{2}$

$\cot\alpha = \frac{3}{x}$

$\tan\alpha = \frac{x}{2}$

$\rightarrow \cot\alpha - \tan\alpha = 2\cot 2\alpha$

$\Rightarrow \frac{3}{x} - \frac{x}{2} = x \rightarrow \frac{9-x^2}{2x} = x \rightarrow 2x^2 = -x^2+9$

$\rightarrow x^2 = 3 \rightarrow x = \frac{2}{\sqrt{3}}$

$\Rightarrow \cot\alpha = \frac{3}{x} = \frac{3}{\frac{2}{\sqrt{3}}} \Rightarrow 2$ ✓

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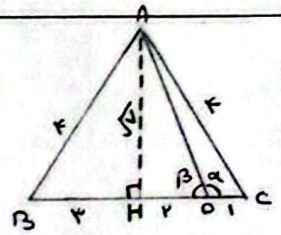

$\hat{AHB}$: $AB^2 = AH^2 + BH^2 \rightarrow AH = \sqrt{12}$

$BC = 4$ (مثلث متساوی الساقین) $\rightarrow BH = CH = 4$

$\tan\beta = \frac{AH}{OH} = \frac{\sqrt{12}}{2}$ (β مکمل α است) $\rightarrow \tan\alpha = -\tan\beta = -\frac{\sqrt{12}}{2}$ ✓

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$2\sin^2 m + \cos^2 m = \frac{5}{3} \rightarrow \sin^2 m + \underbrace{\sin^2 m + \cos^2 m}_{1} = \frac{5}{3} \rightarrow \sin^2 m = \frac{1}{3}$

$\cos^2 m = 1 - \sin^2 m = 1 - \frac{1}{3} = \frac{2}{3}$

$\tan^2 m = \frac{\sin^2 m}{\cos^2 m} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$ ✓

$1-\sin^2\alpha$

$$\frac{\sin^2\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^2\alpha + 4\sin^2\alpha}{1+\sin^2\alpha} = \frac{\sin^2\alpha - 4\sin^2\alpha + 4}{2-\sin^2\alpha} - \frac{\cos^2\alpha - 4\cos^2\alpha + 4}{2-\cos^2\alpha}$$

$1-\sin^2\alpha$ ، $1-\cos^2\alpha$ ، $1-\cos^2\alpha$ ، $1-\sin^2\alpha$

$$= \frac{(\sin^2\alpha-2)^2}{2-\sin^2\alpha} - \frac{(\cos^2\alpha-2)^2}{2-\cos^2\alpha} = -\sin^2\alpha + 2 - 2 + \cos^2\alpha = \cos^2\alpha - \sin^2\alpha = \frac{1+\cos 2\alpha}{2} - \frac{1-\cos 2\alpha}{2}$$

$$= \boxed{\cos 2\alpha}$$

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$\tan\alpha = \frac{4}{3}$ ربع سوم α $\Rightarrow$ $\cos^2\alpha = \frac{1}{1+\frac{16}{9}} = \frac{9}{25} \Rightarrow \cos\alpha = \frac{-3}{5}$ ، $\sin\alpha = \frac{-4}{5}$

$$\frac{\sin\left(\frac{9\pi}{2}+\alpha\right)}{\cos\alpha}\frac{\cos\left(\frac{7\pi}{2}-\alpha\right)}{-\sin\alpha} - \frac{\tan\left(\alpha-\frac{3\pi}{2}\right)}{-\cot\alpha} = -\sin\alpha\cos\alpha + \cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{\cos^2\alpha}{\cos\alpha(1-\sin^2\alpha)}{\sin\alpha}$$

$$\Rightarrow \frac{\cos^2\alpha}{\sin\alpha} = \frac{\left(\frac{-3}{5}\right)^2}{\frac{-4}{5}} = \boxed{\frac{27}{100}}$$

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$\alpha = \frac{\pi}{12} = 15^\circ$

$$\underbrace{2\cos 4\alpha}_{2\cos\frac{\pi}{3}} + \underbrace{\sqrt{2}\sin\alpha - \sqrt{2}\cos\alpha}_{\sqrt{2}(\sin\alpha-\cos\alpha)\ \sqrt{2}\sin\left(\alpha-\frac{\pi}{4}\right)} = 2\times\cos 60^\circ + 2\times\sin(-30^\circ) = 2\times\frac{1}{2} - 2\times\frac{1}{2} = \boxed{\frac{1}{2}}$$

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$\tan\left(\frac{\alpha}{2}\right) = \frac{1}{4}$

$\tan\alpha = \frac{2\tan\left(\frac{\alpha}{2}\right)}{1-\tan^2\left(\frac{\alpha}{2}\right)} = \frac{2\times\frac{1}{4}}{1-\frac{1}{16}} = \frac{8}{15}$

هر دو زاویه در ربع اول $\Rightarrow \sin\alpha, \cos\alpha > 0$

$\cos\alpha = \frac{15}{17}$ ، $\sin\alpha = \frac{8}{17}$

$$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{\frac{8(17-15)}{15\times17}}{\frac{-7}{17}} = \boxed{\frac{-16}{105}}$$

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$\frac{\cot\alpha}{\sin\alpha} > 0 \Rightarrow \frac{\cos\alpha}{\sin^2\alpha} > 0 \Rightarrow \cos\alpha > 0 \Rightarrow$ ربع اول و چهارم ①

$2\sin\alpha < \sin 2\alpha \Rightarrow 2\sin\alpha < 2\sin\alpha\cos\alpha \Rightarrow \sin\alpha\cos\alpha - \sin\alpha > 0 \Rightarrow \sin\alpha(\underbrace{\cos\alpha - 1}_{-1+\cos\alpha<0}) > 0$

$\Rightarrow \sin\alpha < 0 \Rightarrow$ ربع سوم و چهارم ②

① و ② $\Rightarrow$ α در ربع چهارم قرار دارد

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