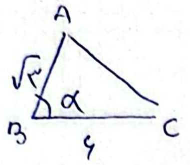
۲۰

نام و نام خانوادگی ....هما عطایی.... پاسخنامه تشریحی تکلیف شماره ۲.۷... کلاس ....یازدهم ریاضی C....

۱

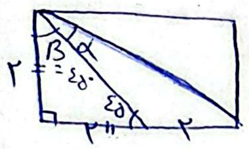


$S=\frac{1}{۲}AB\times BC\times \sin\alpha = ۴/۵$

$\Rightarrow S=\frac{1}{۲}\times\sqrt{۳}\times ۶\times\sin\alpha = ۴/۵ \to \sin\alpha=\frac{۴/۵\times ۲}{۶\times\sqrt{۳}}=\frac{\sqrt{۳}}{۲}$

$\sin\alpha=\frac{\sqrt{۳}}{۲}\begin{cases}\alpha_{max}=\frac{۲\pi}{۳}\\ \alpha_{min}=\frac{\pi}{۳}\end{cases}\to \frac{\alpha_{max}}{\alpha_{min}}=\frac{\frac{۲\pi}{۳}}{\frac{\pi}{۳}}=۲$ جواب

۲



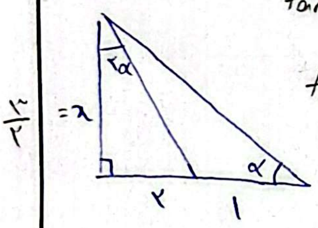
$\beta=۴۵^\circ \Rightarrow \tan\beta=\tan ۴۵=1$ $\tan(\alpha+\beta)=\frac{۴}{۲}=۲$

$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}\Rightarrow ۲=\frac{\tan\alpha+1}{1-\tan\alpha}$

$۲-۲\tan\alpha=\tan\alpha+1\to 1=۳\tan\alpha\to\tan\alpha=\frac{1}{۳}\Rightarrow \cot\alpha=۳$

۳

$\cot\alpha=?$



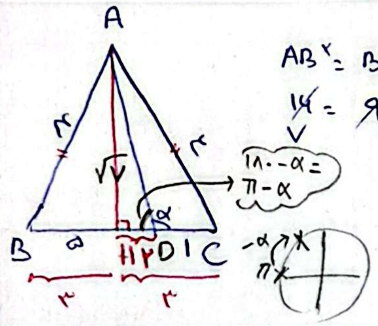
$\tan\alpha=\frac{x}{۳}$, $\tan ۲\alpha=\frac{۲}{x}\Rightarrow \tan ۲\alpha=\frac{۲\tan\alpha}{1-\tan^۲\alpha}=\frac{\frac{۲x}{۳}}{1-\frac{x^۲}{۹}}=\frac{۶x}{۹-x^۲}$

$\left.\begin{array}{l}\tan ۲\alpha=\frac{۶x}{۹-x^۲}\\ \tan ۲\alpha=\frac{۲}{x}\end{array}\right\}\Rightarrow \frac{۶x}{۹-x^۲}=\frac{۲}{x}\to ۳x^۲=۹-x^۲\to ۴x^۲=۹\to x=\pm\frac{۳}{۲}\to x=\frac{۳}{۲}$

غق

$\cot\alpha=\frac{۳}{x}=\frac{۳}{\frac{۳}{۲}}=۲$

۴



$AB^۲=BH^۲+AH^۲$

$۱۶=۹+AH^۲\to AH=\sqrt{۷}$

$۱۸۰-\alpha=\pi-\alpha$

$\tan(\pi-\alpha)=\frac{AH}{HD}=\frac{\sqrt{۷}}{۲}$

$\tan(\pi-\alpha)=-\tan\alpha=\frac{\sqrt{۷}}{۲}\to \tan\alpha=\frac{-\sqrt{۷}}{۲}$ جواب

۵

$۲\sin^۲x+\cos^۲x=\frac{۴}{۳}\to \sin^۲x+\frac{۳}{۳}=\frac{۴}{۳}\to \sin^۲x=\frac{1}{۳}$ جایگذاری $\Rightarrow ۲\left(\frac{1}{۳}\right)+\cos^۲x=\frac{۴}{۳}$

$\sin^۲x+\cos^۲x=1$ $\Rightarrow \cos^۲x=\frac{۲}{۳}$

$\tan^۲x=\frac{\sin^۲x}{\cos^۲x}=\frac{\frac{1}{۳}}{\frac{۲}{۳}}=\frac{1}{۲}$ جواب

$(\sin^2\alpha)^2 = (1-\cos^2\alpha)^2 \qquad (\cos^2\alpha)^2 = (1-\sin^2\alpha)^2$

$$\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha} = \frac{(1-\cos^2\alpha)^2 + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{(1-\sin^2\alpha)^2 + 4\sin^2\alpha}{1+\sin^2\alpha}$$ ۶

$$= \frac{1+\cos^4\alpha - 2\cos^2\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{1+\sin^4\alpha - 2\sin^2\alpha + 4\sin^2\alpha}{1+\sin^2\alpha} = \frac{(\cos^2\alpha+1)^2}{\cos^2\alpha+1} - \frac{(\sin^2\alpha+1)^2}{\sin^2\alpha+1} = \cos^2\alpha + \cancel{1} - \sin^2\alpha - \cancel{1}$$ ۴

جواب

$= \cos^2\alpha - \sin^2\alpha = \cos 2\alpha$

$$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right) - \tan\left(\alpha - \frac{3\pi}{2}\right) = \sin\left(\frac{\pi}{2}+\alpha\right)\cos\left(\frac{3\pi}{2}-\alpha\right) + \tan\left(\frac{3\pi}{2}-\alpha\right) \xrightarrow{\text{روش ضمنی نسبتم } \alpha \text{ زاویه } 90^\circ}$$ ۷

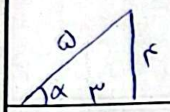
$\frac{8\pi}{2}+\frac{\pi}{2}$, $4\pi = 0$

$\frac{6\pi}{2}+\frac{\pi}{2} = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$, $3\pi = \pi$

$$= \cos(\alpha)(-\sin(\alpha)) + \cot(\alpha) = \left(-\frac{3}{5}\right)\left(-\underbrace{\left(-\frac{4}{5}\right)}_{\frac{4}{5}}\right) + \left(\frac{3}{4}\right)$$

$$= -\frac{12}{25} + \frac{3}{4} = \frac{-48+75}{100} = \frac{27}{100} = 0/27$$ ۴

جواب



$\tan\alpha = \frac{4}{3} \Rightarrow \begin{cases} \cos\alpha = \pm\frac{3}{5} \Rightarrow \cos\alpha = -\frac{3}{5} \\ \sin\alpha = \pm\frac{4}{5} \Rightarrow \sin\alpha = -\frac{4}{5} \end{cases}$, $\cot\alpha = \frac{3}{4}$

زاویه α در ربع ۳ — منفی — منفی — با توجه به α در ربع سوم

$\left.\begin{matrix} \sin\alpha < 0 \\ \tan\alpha, \cot\alpha > 0 \\ \cos\alpha < 0 \end{matrix}\right\} \Leftarrow$ α در ربع سوم

$x = \frac{\pi}{12}$

$$\left(3\cos 4x + \sqrt{2}\sin x - \sqrt{2}\cos x\right) = \frac{3}{2} + \sqrt{2}(\sin x - \cos x) = \frac{3}{2} + \sqrt{2}\left(-\sqrt{1-\sin 2x}\right)$$

$$3\cos 4x = 3\cos\frac{\pi}{3} = 3\times\frac{1}{2} = \frac{3}{2}$$ ۸

$$\Rightarrow \frac{3}{2} + \sqrt{2}\left(-\sqrt{\underbrace{1-\sin\frac{\pi}{6}}_{1-\frac{1}{2} = \frac{1}{2}}}\right) \Rightarrow \frac{3}{2} + \cancel{\sqrt{2}}\left(-\frac{1}{\cancel{\sqrt{2}}}\right) = \frac{3}{2} - 1 = \frac{1}{2}$$ ۲

جواب نهایی

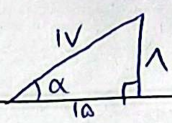
$\tan\frac{\alpha}{2} = \frac{1}{4}$

$$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{\frac{8(17-15)}{15\times\cancel{17}}}{\frac{-7}{\cancel{17}}} = \frac{-16}{105}$$

جواب نهایی

$$\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} = \frac{2\times\frac{1}{4}}{1-\frac{1}{16}} = \frac{\frac{1}{2}}{\frac{15}{16}} = \frac{8}{15}$$ ۱ ۹

$$\cos\alpha = \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \frac{1-\frac{1}{16}}{1+\frac{1}{16}} = \frac{\frac{15}{16}}{\frac{17}{16}} = \frac{15}{17}$$



$\sin\alpha = \frac{8}{17}$

$$\frac{\cot\alpha}{\sin\alpha} > 0 \Rightarrow \frac{\cos\alpha}{\underbrace{\sin^2\alpha}_{\neq 0}} > 0 \Rightarrow \cos\alpha$$

چون $\sin^2\alpha$ همواره + است و عبارت بزرگتر از صفر است پس $\cos\alpha$ نیز + می‌باشد.

$\Rightarrow \cos\alpha > 0$ ناحیه اول یا ناحیه چهارم

همواره مثبت +

$2\sin\alpha\cos\alpha$

از آنجایی که $\cos\alpha > 0$ و $\sin\alpha < 0$ در ناحیه چهارم مثلثاتی α قرار دارد ۴ ۱۰

$$2\sin\alpha < \sin 2\alpha \Rightarrow 2\sin\alpha - 2\sin\alpha\cos\alpha < 0 \Rightarrow 2\sin\alpha\,\overbrace{(1-\cos\alpha)}^{+} < 0 \Rightarrow \sin\alpha < 0$$

$0 < \cos\alpha < 1$

ناحیه چهارم