نام و نام خانوادگی ......مهسا عطایی...... پاسخنامه تشریحی تکلیف شماره ۲۰۷... کلاس ......یازدهم ریاضی......

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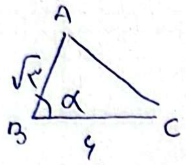

$$S = \frac{1}{2} AB \times BC \times \sin\alpha = 4.5$$

$$\Rightarrow S = \frac{1}{2} \times \sqrt{3} \times 6 \times \sin\alpha = 4.5 \rightarrow \sin\alpha = \frac{\cancel{4.5}^{9} \times \cancel{2}}{6 \times \sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\sin\alpha = \frac{\sqrt{3}}{2} \begin{cases} \alpha_{max} = \frac{2\pi}{3} \\ \alpha_{min} = \frac{\pi}{3} \end{cases} \rightarrow \frac{\alpha_{max}}{\alpha_{min}} = \frac{\frac{2\pi}{3}}{\frac{\pi}{3}} = 2$$

جواب

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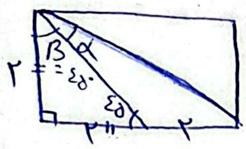

$$\beta = 45^\circ \Rightarrow \tan\beta = \tan 45 = 1 \qquad \tan(\alpha+\beta) = \frac{4}{2} = 2$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta} \Rightarrow 2 = \frac{\tan\alpha + 1}{1 - \tan\alpha}$$

$$\cancel{2} - 2\tan\alpha = \tan\alpha + \cancel{1} \rightarrow 1 = 3\tan\alpha \rightarrow \tan\alpha = \frac{1}{3} \Rightarrow \cot\alpha = 3$$

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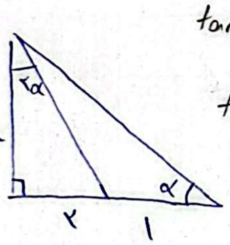
$\cot\alpha = ?$

$\frac{3}{2} = x$



$$\tan\alpha = \frac{x}{3}, \quad \tan 2\alpha = \frac{2}{x} \Rightarrow \tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha} = \frac{\frac{2x}{3}}{1 - \frac{x^2}{9}} = \frac{\frac{2x}{\cancel{3}}}{\frac{9 - x^2}{\cancel{9}_{3}}}$$

$$\left.\begin{array}{l} \tan 2\alpha = \frac{6x}{9 - x^2} \\ \tan 2\alpha = \frac{2}{x} \end{array}\right\} \Rightarrow \frac{\cancel{6}^{3}x}{9 - x^2} = \frac{\cancel{2}}{x} \rightarrow 3x^2 = 9 - x^2 \rightarrow 4x^2 = 9 \rightarrow x = \pm\frac{3}{2} \rightarrow x = \frac{3}{2}$$

غ ق

$$\cot\alpha = \frac{3}{x} = \frac{\cancel{3}}{\frac{\cancel{3}}{2}} = 2$$

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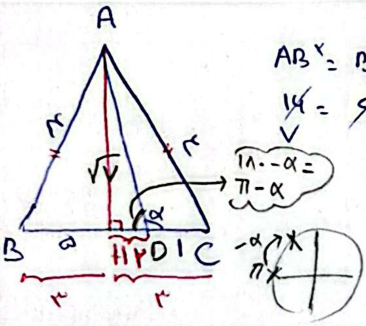

$$AB^2 = BH^2 + AH^2$$

$$16 = 9 + AH^2 \rightarrow AH = \sqrt{7}$$

$$180 - \alpha = \pi - \alpha$$

$$\tan(\pi - \alpha) = \frac{AH}{HD} = \frac{\sqrt{7}}{2}$$

جواب

$$\tan(\pi - \alpha) = -\tan\alpha = \frac{\sqrt{7}}{2} \rightarrow \tan\alpha = \frac{-\sqrt{7}}{2}$$

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$$2\sin^2 x + \cos^2 x = \frac{4}{3} \rightarrow \sin^2 x + \frac{3}{3} = \frac{4}{3} \rightarrow \sin^2 x = \frac{1}{3}$$

جایگذاری

$$\Rightarrow 2\left(\frac{1}{3}\right) + \cos^2 x = \frac{4}{3} \Rightarrow \cos^2 x = \frac{2}{3}$$

$$\sin^2 x + \cos^2 x = 1$$

جواب

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{1}{\cancel{3}}}{\frac{2}{\cancel{3}}} = \frac{1}{2}$$

$(\sin^2\alpha)^2 = (1-\cos^2\alpha)^2 \qquad (\cos^2\alpha)^2 = (1-\sin^2\alpha)^2$

$$\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha} = \frac{(1-\cos^2\alpha)^2 + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{(1-\sin^2\alpha)^2 + 4\sin^2\alpha}{1+\sin^2\alpha}$$ 6

$$= \frac{1+\cos^4\alpha - \cancel{2\cos^2\alpha} + \cancel{4\cos^2\alpha}}{1+\cos^2\alpha} - \frac{1+\sin^4\alpha - \cancel{2\sin^2\alpha} + \cancel{4\sin^2\alpha}}{1+\sin^2\alpha} = \frac{(\cos^2\alpha+1)^2}{\cos^2\alpha+1} - \frac{(\sin^2\alpha+1)^2}{\sin^2\alpha+1} = \cos^2\alpha + \cancel{1} - \sin^2\alpha - \cancel{1}$$

جواب

$$= \cos^2\alpha - \sin^2\alpha = \cos 2\alpha$$

$$\sin\left(\frac{13\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right) - \tan\left(\alpha - \frac{3\pi}{2}\right) = \sin\left(\frac{\pi}{2}+\alpha\right)\cos\left(3\frac{\pi}{2}-\alpha\right) + \tan\left(3\frac{\pi}{2}-\alpha\right) \xrightarrow{\text{زوج بودن نسبت } \alpha \text{ و } 90^\circ}$$

$\frac{12\pi}{2}+\frac{\pi}{2}$, $4\pi = 0$

$\frac{6\pi}{2}+\frac{\pi}{2} = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$, $3\pi = \pi$

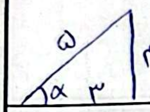
$$= \cos(\alpha)(-\sin(\alpha)) + \cot(\alpha) = \left(-\frac{3}{5}\right)\left(-\left(-\frac{4}{5}\right)\right) + \left(\frac{3}{4}\right)$$

$$= -\frac{12}{25} + \frac{3}{4} = \frac{-48+75}{100} = \frac{27}{100} = 0.27$$

جواب

$\tan\alpha = \frac{4}{3} \Rightarrow \begin{cases} \cos\alpha = \pm\frac{3}{5} \Rightarrow \cos\alpha = -\frac{3}{5} \\ \sin\alpha = \pm\frac{4}{5} \Rightarrow \sin\alpha = -\frac{4}{5} \end{cases}$, $\cot\alpha = \frac{3}{4}$

منفی — چون α در ربع سوم



$\left.\begin{matrix} \sin\alpha < 0 \\ \tan\alpha, \cot\alpha > 0 \\ \cos\alpha < 0 \end{matrix}\right\} \Rightarrow$ α در ربع سوم

$x = \frac{\pi}{12}$

$$\left(3\cos 4x + \sqrt{2}\sin x - \sqrt{2}\cos x\right) = \frac{3}{2} + \sqrt{2}(\sin x - \cos x) = \frac{3}{2} + \sqrt{2}\left(-\sqrt{1-\sin 2x}\right)$$

$3\cos 4x = 3\cos\frac{\pi}{3} = 3\times\frac{1}{2} = \frac{3}{2}$

$$\Rightarrow \frac{3}{2} + \sqrt{2}\left(-\sqrt{1-\underbrace{\sin\frac{\pi}{6}}_{\frac{1}{2}}}\right) \Rightarrow \frac{3}{2} + \cancel{\sqrt{2}}\left(-\frac{1}{\cancel{\sqrt{2}}}\right) = \frac{3}{2} - 1 = \frac{1}{2}$$ 8

جواب نهایی

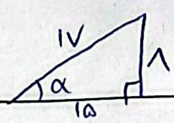
$\tan\frac{\alpha}{2} = \frac{1}{4}$

$$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{\frac{8(17-15)}{15\times 17}}{\frac{-7}{17}} = \frac{-16}{105}$$

جواب نهایی

$$\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} = \frac{2\times\frac{1}{4}}{1-\frac{1}{16}} = \frac{\frac{1}{2}}{\frac{15}{16}} = \frac{8}{15}$$ 9

$$\cos\alpha = \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \frac{1-\frac{1}{16}}{1+\frac{1}{16}} = \frac{\frac{15}{16}}{\frac{17}{16}} = \frac{15}{17}$$



$\sin\alpha = \frac{8}{17}$

$$\frac{\cot\alpha}{\sin\alpha} > 0 \Rightarrow \frac{\cos\alpha}{\underset{+}{\sin^2\alpha}} > 0 \Rightarrow \cos\alpha \Rightarrow \cos\alpha > 0$$

چون $\sin^2\alpha$ همواره + است و عبارت بزرگتر از صفر است پس $\cos\alpha$ نیز + می‌باشد.

ناحیه اول یا ناحیه چهارم

از آنجایی که α در ناحیه چهارم مشترک قرار دارد $\cos\alpha > 0$, $\sin\alpha < 0$ 10

همواره مثبت +

$$2\sin\alpha < \sin 2\alpha \Rightarrow 2\sin\alpha - \underbrace{2\sin\alpha\cos\alpha} < 0 \Rightarrow 2\sin\alpha\left(\overset{+}{1-\cos\alpha}\right) < 0 \Rightarrow \sin\alpha < 0$$

$0 < \cos\alpha < 1$

ناحیه چهارم