زهرا صدرالعلمایی یازدهم ریاضی C تکلیف ۲۷ (۲۰)

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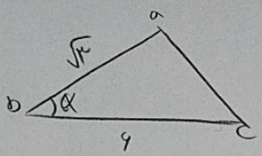

$S=\frac{1}{2}\times ab\times bc\times \sin\alpha = \frac{1}{2}\times 4\times 6\sqrt{2}\times \sin\alpha = \frac{\sqrt{3}}{...}$

$\sin\alpha=\frac{\sqrt{3}}{2} \Rightarrow \alpha=60^\circ , 120^\circ \qquad \frac{120^\circ}{60^\circ}=$ ۲ برابر

(۲)

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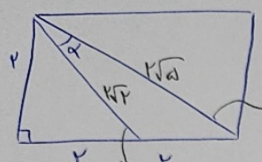
$\cot\alpha = ?$



$\sqrt{2^2+4^2}=\sqrt{4+16}=2\sqrt{5}$

$\sqrt{2^2+2^2}=\sqrt{8}$

$c^2=a^2+b^2-2ab\cos\alpha$

$4=8+20-2\times 2\sqrt{2}\times 2\sqrt{5}\times\cos\alpha$ ($4=28-8\sqrt{10}\times\cos\alpha$)

$24=8\sqrt{10}\cos\alpha$

$\frac{3}{\sqrt{10}}=\cos\alpha \longrightarrow \sin\alpha=\frac{1}{\sqrt{10}}$

$\cot^2\alpha+1=\frac{1}{\sin^2\alpha}=\frac{1}{\frac{1}{10}}=10 \longrightarrow \cot^2\alpha=9$

$\cot\alpha=3$ (زاویه تند)

(۲)

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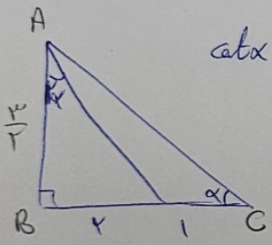

$\cot\alpha = ?$

$\tan 2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}=\frac{3}{AB}=\frac{2\times\frac{AB}{3}}{1-\frac{AB^2}{9}}=\frac{AB^2}{3}=\frac{9-AB^2}{3}$

(۲)

$3AB^2=9-AB^2 \longrightarrow 4AB^2=9 \qquad AB=\frac{3}{2}$

$\cot\alpha=\frac{\frac{3}{1}}{\frac{3}{2}}=2$

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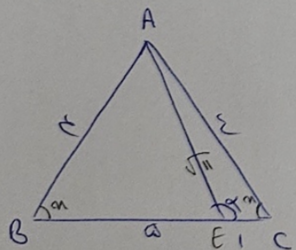

$AE^2=16+1-8\cos x=16+25-40\cos x$

$-\cos x=3-5\cos x \qquad 4\cos x=3 \qquad \cos x=\frac{3}{4}$

$16+1-8\times\frac{3}{4}=16+1-6=11 \rightarrow AE=\sqrt{11}$

$16=11+25-2\sqrt{11}\cdot 5\cos\alpha$

$\frac{9}{2}=-10\sqrt{11}\cos\alpha = \frac{-20}{\sqrt{11}}=\cos\alpha \rightarrow 1+\tan^2\alpha=\frac{1}{\frac{4}{11}}=\frac{11}{4}$

$\tan^2\alpha=\frac{7}{4} \rightarrow \tan\alpha=\frac{-\sqrt{7}}{2}$

(۲)

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$2\sin^2x+\cos^2x=\frac{4}{3} = \sin^2x+1=\frac{4}{3} \rightarrow \sin^2x=\frac{1}{3} \longrightarrow \sin^2x+\cos^2x=1$ ($\cos^2x=\frac{2}{3}$)

$\tan^2x=? \qquad 1+\tan^2x=\frac{1}{\cos^2x=\frac{2}{3}}=\frac{3}{2} \rightarrow \tan^2x=\frac{1}{2}$

(۲)

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$$\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha} \qquad (1-\sin^2\alpha \text{ و } 1-\cos^2\alpha)$$

$$\frac{\sin^4\alpha + 4 - 4\sin^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4 - 4\cos^2\alpha}{1+\sin^2\alpha}$$

$$(\cos^2\alpha+1)^2 = (1-\cos^2\alpha-2)^2 \qquad (\sin^2\alpha+1)^2 = (1-\sin^2\alpha-2)^2$$

$$= \frac{(\sin^2\alpha-2)^2}{1+\cos^2\alpha} - \frac{(\cos^2\alpha-2)^2}{1+\sin^2\alpha}$$

$$= \cos^2\alpha+1 - 1+\sin^2\alpha = \cos^2\alpha-\sin^2\alpha = \boxed{\cos 2\alpha}$$

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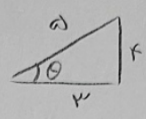
$$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right) + \tan\left(\frac{13\pi}{2}-\alpha\right) - \tan\left(\alpha-\frac{11\pi}{2}\right)$$



$$(+\cos\alpha)(-\sin\alpha) + (\cot\alpha)$$

$$\left(-\frac{3}{5}\right)\left(+\frac{4}{5}\right) + \left(\frac{3}{4}\right) = -\frac{12}{25} + \frac{3}{4} = \frac{-48+75}{100} = \boxed{\frac{27}{100}}$$

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$$\left(2\cos 4x + \sqrt{2}\sin x - \sqrt{2}\cos 3x\right) \qquad x = \frac{\pi}{12}$$

$$\sin^2\frac{\pi}{12} = \frac{1-\cos\frac{\pi}{6}}{2} = \frac{1-\frac{\sqrt{3}}{2}}{2} = \frac{2-\sqrt{3}}{4} \Rightarrow \frac{\sqrt{2-\sqrt{3}}}{2} = \sin\frac{\pi}{12}$$

$$2\cos\frac{\pi}{3} + \sqrt{2}\sin\frac{\pi}{12} - \sqrt{2}\cos\frac{3\pi}{12}$$

$$\frac{2}{2} + \sqrt{2}\left(\sin\frac{\pi}{12} - \cos\frac{\pi}{12}\right) = \frac{2}{2} + \sqrt{2}\left(\frac{\sqrt{2-\sqrt{3}}}{2} - \frac{\sqrt{2+\sqrt{3}}}{2}\right)$$

$$= \frac{2}{2} + \left(\frac{\sqrt{6-2\sqrt{9}}}{2} - \frac{\sqrt{4+2\sqrt{3}}}{2}\right) = \frac{2}{2} + \frac{\sqrt{(\sqrt{3}-1)^2} - \sqrt{(\sqrt{3}+1)^2}}{2}$$

$$= \frac{2}{2} + \frac{(\sqrt{3}-1) - (\sqrt{3}+1)}{2} = \frac{2}{2} + \frac{-2}{2} = \boxed{\frac{1}{2}}$$

۹-

$$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} \qquad \tan\left(\frac{\alpha}{2}\right) = \frac{1}{4} \rightarrow \tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} = \frac{2\times\frac{1}{4}}{1-\frac{1}{16}} = \frac{\frac{1}{2}}{\frac{15}{16}} = \frac{8}{15}$$

$$\frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{\frac{136-120}{255}}{-\frac{7}{17}} = \frac{16}{-105}$$

چون $\frac{\alpha}{2}$ و $\tan\alpha$ هردو مثبت هستند، $\tan\alpha$ هم مثبت است.

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$$2\sin\alpha < \sin 2\alpha \rightarrow 2\sin\alpha < 2\sin\alpha\cos\alpha \qquad \sin\alpha - \sin\alpha\cos\alpha < 0$$

$$\sin\alpha(1-\cos\alpha) < 0 \Rightarrow \boxed{\sin\alpha < 0}$$

$1-\cos\alpha > 0$ همیشه مثبت

$$0 < \frac{\cot\alpha}{\sin\alpha} = \frac{\cos\alpha}{\sin^2\alpha} \Rightarrow \boxed{\cos\alpha > 0}$$

چون α ناحیه ۴ است.