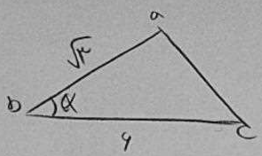
رزانه صدرالعلمایی   یازدهم دختر C   تکلیف ۲۷

۱-

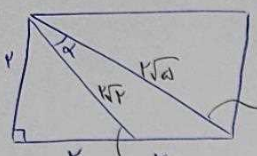


$$S=\frac{1}{2}\times ab\times bc\times \sin\alpha=\frac{1}{\cancel{2}}\times\cancel{6}^{3}\times\cancel{\sqrt{4}}\times\sin\alpha=\frac{\sqrt{3}}{\cancel{2}}$$

$$\sin\alpha=\frac{\sqrt{3}}{2}\Rightarrow\alpha=60^\circ,120^\circ \qquad \frac{120^\circ}{60^\circ}=\boxed{2\ \text{برابر}}$$

۲-

$\cot\alpha=?$



$$\sqrt{2^2+4^2}=\sqrt{4+16}=2\sqrt{5} \qquad \sqrt{2^2+2^2}=\sqrt{8}$$

$$c^2=a^2+b^2-2ab\cos\alpha$$

$$4=\overset{28}{8+20}-2\times4\sqrt{10}\times\cos\alpha$$

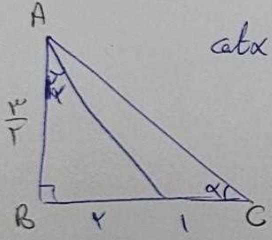
$$\overset{3}{\cancel{24}}=\cancel{8}\sqrt{10}\cos\alpha$$

$$\frac{3}{\sqrt{10}}=\cos\alpha\longrightarrow\sin\alpha=\frac{1}{\sqrt{10}}$$

$$\cot^2\alpha+1=\frac{1}{\sin^2\alpha}=\frac{1}{\frac{1}{10}}=10\rightarrow\cot^2\alpha=9 \qquad \boxed{\cot\alpha=3}$$

زاویه تیز

۳-



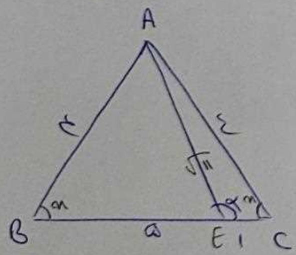
$\cot\alpha=?$

$$\tan2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}=\frac{2}{AB}=\frac{2\times\frac{AB}{3}}{1-\frac{AB^2}{9}}=\frac{AB^2}{2}=\frac{9-AB^2}{\frac{\cancel{9}}{3}}$$

$$3AB^2=9-AB^2\longrightarrow4AB^2=9 \qquad AB=\frac{3}{2}$$

$$\cot\alpha=\frac{\frac{3}{1}}{\frac{3}{2}}=2$$

۴-



$$AE^2=16+1-8\cos x=16+25-40\cos x$$

$$-\cos x=3-5\cos x \qquad 4\cos x=3 \qquad \cos x=\frac{3}{4}$$

$$16+1-\underbrace{8\times\frac{3}{4}}_{6}=16+1-6=11\rightarrow AE=\sqrt{11}$$

$$\cancel{16}=11+1-2\sqrt{11}\cos\alpha$$

$$\frac{4}{2}=-2\sqrt{11}\cos\alpha=-\frac{2}{\sqrt{11}}=\cos\alpha\longrightarrow1+\tan^2\alpha=\frac{1}{\frac{4}{11}}=\frac{11}{4}$$

$$\tan^2\alpha=\frac{7}{4}\rightarrow\boxed{\tan\alpha=-\frac{\sqrt{7}}{2}}$$

۵-

$$2\sin^2x+\cos^2x=\frac{4}{3}=\sin^2x+1=\frac{4}{3}\rightarrow\sin^2x=\frac{1}{3}\longrightarrow\sin^2x+\cos^2x=1$$

$$\cos^2x=\frac{2}{3}$$

$$\tan^2x=? \qquad 1+\tan^2x=\frac{1}{\cos^2x=\frac{2}{3}}=\frac{3}{2}\rightarrow\boxed{\tan^2x=\frac{1}{2}}$$

-۶

$$\frac{\sin^4\alpha + 4\overset{1-\sin^2\alpha}{\cos^2\alpha}}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4\overset{1-\cos^2\alpha}{\sin^2\alpha}}{1+\sin^2\alpha} \qquad \frac{\sin^4\alpha+4-4\sin^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha+4-4\cos^2\alpha}{1+\sin^2\alpha}$$

$$(\cos^2\alpha+1)^2 = (1-\cos^2\alpha-2)^2 \qquad = \frac{(\sin^2\alpha-2)^2}{1+\cos^2\alpha} - \frac{(\cos^2\alpha-2)^2}{1+\sin^2\alpha} \qquad (\cos^2\alpha-2)^2 = (1-\sin^2\alpha-2)^2 = (\sin^2\alpha+1)^2$$

$$= \cos^2\alpha+1 - 1-\sin^2\alpha = \cos^2\alpha-\sin^2\alpha = \boxed{\cos 2\alpha}$$

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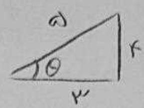
$$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right) \underset{}{-} \tan\left(\alpha-\frac{3\pi}{2}\right) \qquad + \tan\left(\frac{3\pi}{2}-\alpha\right)$$



$$(+\cos\alpha)(-\sin\alpha) + (\cot\alpha)$$

$$\left(-\frac{3}{5}\right)\left(+\frac{4}{5}\right) + \left(\frac{3}{4}\right) = -\frac{12}{25} + \frac{3}{4} = \frac{-48+75}{100} = \boxed{\frac{27}{100}}$$

-۸

$$(3\cos 4x + \sqrt{2}\sin x - \sqrt{2}\cos x) \qquad x = \frac{\pi}{12}$$

$$\sin\frac{\pi}{12} = \sqrt{\frac{1-\overset{\frac{\sqrt3}{2}}{\cos\frac{\pi}{6}}}{2}} = \sqrt{\frac{2-\sqrt3}{4}} = \underbrace{\frac{\sqrt{2-\sqrt3}}{2}}_{\sin\frac{\pi}{12}}$$

$$3\cos\frac{\pi}{3} + \sqrt2\sin\frac{\pi}{12} - \sqrt2\cos\frac{\pi}{12}$$

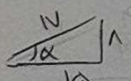
$$\frac{3}{2} + \sqrt2\left(\sin\frac{\pi}{12} - \cos\frac{\pi}{12}\right) = \frac32 + \sqrt2\left(\frac{\sqrt{2-\sqrt3}}{2} - \frac{\sqrt{2+\sqrt3}}{2}\right)$$

$$= \frac32 + \left(\frac{\sqrt{4-2\sqrt3}}{2} - \frac{\sqrt{4+2\sqrt3}}{2}\right) = \frac32 + \frac{\overbrace{\cancel{\sqrt3}-1-\cancel{\sqrt3}-1}^{-2}}{2} = \boxed{\frac12}$$

$$\frac{\sqrt{(\sqrt3-1)^2}}{2} - \frac{\sqrt{(\sqrt3+1)^2}}{2}$$

-۹

$$\frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha} \qquad \tan\left(\frac{\alpha}{2}\right) = \frac14 \longrightarrow \tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} = \frac{2\times\frac14}{1-\frac{1}{16}} = \frac{\frac12}{\frac{15}{\cancel{16}_{8}}} = \frac{8}{15}$$



$$\frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}} = \frac{\frac{\overset{2\times8}{17\times8-15\times8}}{15\times\cancel{17}}}{\frac{-7}{\cancel{17}}} = \boxed{\frac{16}{-105}}$$

چون $\tan\frac{\alpha}{2}$ و $\tan\alpha$ هر دو مثبت‌اند پس در ناحیه اول بوده‌اند.

-۱۰

$$2\sin\alpha < \sin2\alpha \longrightarrow \cancel{2}\sin\alpha < \cancel{2}\sin\alpha\cos\alpha \qquad \sin\alpha - \sin\alpha\cos\alpha < 0$$

$$\sin\alpha\,\underbrace{(1-\cos\alpha)}_{\underbrace{0<\quad<1}_{\text{همیشه مثبت}}} < 0 \Rightarrow \boxed{\sin\alpha<0}$$

$$0 < \frac{\cot\alpha}{\sin\alpha} = \frac{\cos\alpha}{\underset{+}{\sin^2\alpha}} > 0 \qquad \boxed{\cos\alpha>0}$$

<= کمان α ناحیه ۴ است.