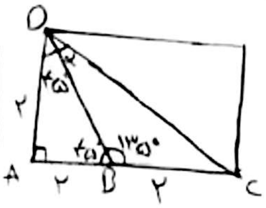
یاران میرسعا ولایتی - تکلیف شماره ۲۷ - یازدهم دختر

-۱

$$\frac{1}{2}\times 6\times\sqrt{3}\times\sin\alpha=4.5 \longrightarrow \sin\alpha=\frac{\sqrt{3}}{2}$$ کمترین $\alpha=60^\circ$ ، بیشترین $\alpha=120^\circ$ $\Longrightarrow$ ۲ برابر

-۲



$$DB=\sqrt{2^2+2^2}=2\sqrt{2}\ ,\ DC=\sqrt{4+16}=2\sqrt{5}$$

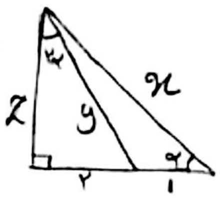
$$S_{BDC}=S_{CDB}\Rightarrow \frac{1}{2}\times\underbrace{DC\times BD}_{2\sqrt{2}\times2\sqrt{5}}\times\sin\alpha=\frac{1}{2}\times\underbrace{DB\times BC}_{2\sqrt{2}\times2}\sin\overset{\frac{\sqrt{2}}{2}}{135}$$

$$\Downarrow$$

$$\sin\alpha=\frac{\sqrt{10}}{10}$$

درنهایت: $1+\cot^2\alpha=\frac{1}{\sin^2\alpha}\Rightarrow\boxed{\cot\alpha=3}$ ، $\left(\frac{\sqrt{10}}{10}\right)^2$

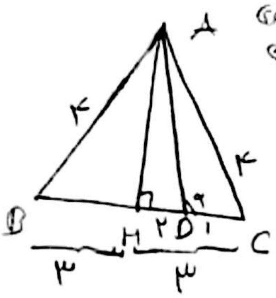
-۳



$$\cos2\alpha=\cos^2\alpha-\sin^2\alpha$$
$$\sin2\alpha=2\sin\alpha\cos\alpha$$
$$\Longrightarrow\begin{cases}\frac{z}{y}=\left(\frac{3}{x}\right)^2-\left(\frac{z}{x}\right)^2\Rightarrow\frac{9-z^2}{x^2}=\frac{z}{y}\\ \frac{2}{y}=2\times\frac{z}{x}\times\frac{3}{x}\Rightarrow x^2=3yz\end{cases}$$ پس

$$\frac{9-z^2}{3yz}=\frac{z}{y}\Rightarrow z=\frac{3}{2}$$ درنهایت: $\cot\alpha=\frac{3}{\underset{\frac{3}{2}}{z}}=\boxed{2}$

-۴



مثلث متساوی الساقین $\to$ میانه = ارتفاع $\to BH=HC=3$

$$AH=\sqrt{16-9}=\sqrt{7}\Longrightarrow AD=\sqrt{(\underbrace{\sqrt{7}}_{7})^2+2^2}=\sqrt{11}$$

$$4=\sqrt{(\sqrt{11})^2+1^2-2\sqrt{11}\cos\alpha}\longrightarrow\cos\alpha=\frac{2\sqrt{11}}{11}$$

$$1+\tan^2\alpha=\frac{1}{\cos^2\alpha}\longrightarrow\underset{\text{زاویه باز}}{\tan\alpha}=-\sqrt{\frac{7}{4}}=\frac{-\sqrt{7}}{2}$$

-۵

$$\sin^2x+\underbrace{\sin^2x+\cos^2x}_{1}=\frac{4}{3}\longrightarrow\sin^2x=\frac{1}{3}\ ,\ \cos^2x=1-\frac{1}{3}=\frac{2}{3}\Longrightarrow$$

$$\tan^2x=\frac{\sin^2x}{\cos^2x}=\frac{\frac{1}{3}}{\frac{2}{3}}=\frac{1}{2}$$

-۶

$$\sin^4\alpha=\underbrace{(1-\cos^2\alpha)^2}_{1-2\cos^2\alpha+\cos^4\alpha}\xrightarrow{+4\cos^2\alpha}\sin^4\alpha+4\cos^2\alpha=\underbrace{1+2\cos^2\alpha+\cos^4\alpha}_{(\cos^2\alpha+1)^2}$$

$$\cos^4\alpha=\underbrace{(1-\sin^2\alpha)^2}_{1-2\sin^2\alpha+\sin^4\alpha}\xrightarrow{+4\sin^2\alpha}\cos^4\alpha+4\sin^2\alpha=\underbrace{1+2\sin^2\alpha+\sin^4\alpha}_{(1+\sin^2\alpha)^2}$$

$$\Longrightarrow$$

$$\frac{(\cos^2\alpha+1)^2}{\cos^2\alpha+1}-\frac{(1+\sin^2\alpha)^2}{1+\sin^2\alpha}\Rightarrow\cos^2\alpha+1-1-\sin^2\alpha=\cos^2\alpha-\sin^2\alpha=\cos2\alpha$$

-۷

یک دور و نیم

$$\sin(۴\pi+\frac{\pi}{۲}+\alpha)\cos(\underbrace{۳\pi+\frac{\pi}{۲}}_{\frac{۳\pi}{۲}}-\alpha)-\tan(\alpha-\frac{۳\pi}{۲})=\cos\alpha(-\sin\alpha)+\cot\alpha$$

دور کامل

$$\underbrace{1+\tan^2\alpha}_{\frac{۱۶}{۹}}=\frac{1}{\cos^2\alpha}\Rightarrow\cos\alpha=-\frac{۳}{۵}\ ,\ \sin\alpha=-\sqrt{1-\frac{۹}{۲۵}}=-\frac{۴}{۵}$$

درنتیجه: $-\frac{۳}{۵}\times\left(-(-\frac{۴}{۵})\right)+\frac{۳}{۴}=\frac{۲۷}{۱۰۰}$

-۸

$$\left.\begin{array}{l}\sqrt{۲}\underbrace{(\sin x-\cos x)}_{\sqrt{۲}\sin(x-\frac{\pi}{۴})}=۲\sin\underbrace{(\frac{\pi}{۱۲}-\frac{\pi}{۴})}_{\frac{-\pi}{۶}}=-1\\ ۳\cos\underbrace{\frac{۴\pi}{۱۲}}_{\frac{\pi}{۳}}=\frac{۳}{۲}\end{array}\right\}\Rightarrow\frac{۳}{۲}-1=\boxed{\frac{1}{۲}}$$

-۹

$$\tan\alpha=\frac{۲\tan(\frac{\alpha}{۲})}{1-\tan^2(\frac{\alpha}{۲})}=\frac{۲\times\frac{1}{۴}}{1-\frac{1}{۱۶}}=\frac{\frac{1}{۲}}{\frac{۱۵}{۱۶}}=\frac{۸}{۱۵}\longrightarrow$$

$\rightarrow x=\sqrt{۶۴+۲۲۵}=۱۷$

$$\frac{\frac{۸}{۱۵}-\frac{۸}{۱۷}}{\frac{۸}{۱۷}-\frac{۱۵}{۱۷}}=-\frac{۱۶}{۱۰۵}$$

-۱۰

$$0<\frac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha}\longrightarrow\frac{\cos\alpha}{\underbrace{\sin^2\alpha}_{+}}>0\longrightarrow\cos\alpha>0$$

$$۲\sin\alpha<\sin۲\alpha\Rightarrow۲\sin\alpha<۲\sin\alpha\cos\alpha\longrightarrow0<\underbrace{۲\sin\alpha}_{\ominus}\underbrace{(\cos\alpha-1)}_{\ominus}$$

$$\begin{cases}\sin\alpha<0\\ \cos\alpha>0\end{cases}\longrightarrow$$ ربع چهارم