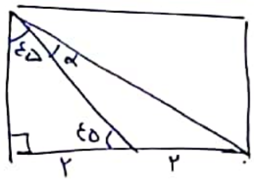
سوال ۱:

$S_{مثلث} = \frac{1}{2} ab\sin\alpha \rightarrow 4{,}5 = \frac{1}{2}\times 6\times\sqrt{3}\times \sin\alpha \rightarrow \sin\alpha = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2} \rightarrow \alpha \begin{cases} 120 \\ 60 \end{cases}$ [۲]

سوال ۲:



$\cot = ?$

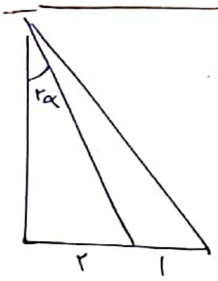
$\cot(\alpha+45) = \frac{2}{4} = \frac{1}{2}$

$\cot(\alpha+45) = \frac{\cot\alpha\cot 45 - 1}{\cot\alpha + \cot 45} = \frac{\cot\alpha \times 1 - 1}{\cot\alpha + 1} = \frac{1}{2}$

$\rightarrow 2\cot\alpha - 2 = \cot\alpha + 1$

$\cot\alpha = 3$

سوال ۳:



$\tan 2\alpha = \frac{2}{x}$

$\tan\alpha = \frac{x}{3}$

$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$

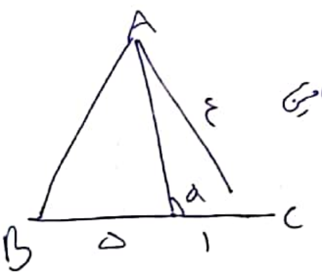
$\rightarrow \frac{2(\frac{x}{3})}{1-(\frac{x^2}{9})} = \frac{2}{x} \rightarrow \frac{\frac{2x}{3}}{\frac{9-x^2}{9}} = \frac{2}{x} \rightarrow \frac{6x}{9-x^2} = \frac{2}{x}$

$6x^2 = 18 - 2x^2 \rightarrow 8x^2 = 18 \quad x^2 = \frac{9}{4}$

$x = \frac{3}{2}$ ← چون طول ضلع نمی‌تواند منفی باشد

$\cot\alpha = \frac{3}{x} = \frac{3}{\frac{3}{2}} = 2$

س ۴:



پایه‌های ارتفاع در مثلث متساوی‌الساقین است

$AH = \sqrt{16-9} = \sqrt{7}$

$\tan(\pi-\alpha) = \frac{\sqrt{7}}{3} \rightarrow -\tan\alpha = \frac{\sqrt{7}}{3} \rightarrow \tan\alpha = -\frac{\sqrt{7}}{3}$

سوال ۵:

$2\sin^2 x + \cos^2 x = \frac{4}{3}, \quad \tan^2 x = ?$

$\sin^2 x + \cos^2 x + \sin^2 x = \frac{4}{3} \rightarrow \sin^2 x = \frac{1}{3} \rightarrow \sin x \begin{cases} \frac{1}{3} \\ -\frac{1}{3} \end{cases}$

$\cos^2 x = 1 - \sin^2 x = 1 - \frac{1}{3} = \frac{2}{3}$

$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$

سوال ۶:

$\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha} =$

$= \frac{\sin^4\alpha + 4(1-\sin^2\alpha)}{2-\sin^2\alpha} - \frac{\cos^4\alpha + 4(1-\cos^2\alpha)}{2-\cos^2\alpha} = \frac{\sin^4\alpha - 4\sin^2\alpha + 4}{2-\sin^2\alpha} - \frac{\cos^4\alpha - 4\cos^2\alpha + 4}{2-\cos^2\alpha}$

$\frac{(\sin^2\alpha - 2)^2}{2-\sin^2\alpha} - \frac{(\cos^2\alpha - 2)^2}{2-\cos^2\alpha} = (2-\sin^2\alpha) - (2-\cos^2\alpha) = \cos^2\alpha - \sin^2\alpha = \cos 2\alpha$

س ۷: $\sin\alpha\cos\alpha<۰$, $\tan\alpha\cot\alpha>۰$

$\tan\alpha=\frac{۴}{۳}$, $\alpha \rightarrow$ ربع سوم

$= \cos\alpha \times (-\sin\alpha) + \cot\alpha$

$= -\frac{۳}{۵} \times \frac{۴}{۵} + \frac{۳}{۴} = \frac{-۱۲}{۲۵} + \frac{۳}{۴} = \frac{-۴۸+۷۵}{۱۰۰} = \frac{۲۷}{۱۰۰}$

س ۸:

$(۳\cos ۴x + \sqrt{۲}\sin x - \sqrt{۲}\cos x) \xrightarrow{x=\frac{\pi}{۱۲}} ? = \frac{۳}{۲} - ۱ = \frac{۱}{۲}$

$\rightarrow (۳\cos\frac{۴\pi}{۱۲} + \sqrt{۲}\sin\frac{\pi}{۱۲} - \sqrt{۲}\cos\frac{\pi}{۱۲})$

$۳ \times \frac{۱}{۲} = \frac{۳}{۲}$ $\quad \sqrt{۲}(\sqrt{۲}\sin(\frac{\pi}{۱۲} - \frac{\pi}{۴})) \xrightarrow{-\frac{\pi}{۶}} -۲\sin\frac{\pi}{۶} = -۱$

س ۹)

$\tan(\frac{\alpha}{2}) = \frac{1}{4}$ $\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = ?$

$\tan\alpha = \frac{2\times\frac{1}{4}}{1-\frac{1}{16}} = \frac{8}{15} \rightarrow \begin{cases} \sin\alpha = \frac{8}{17} \\ \cos\alpha = \frac{15}{17} \end{cases}$

$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{\frac{16}{15\times17}}{\frac{-7}{17}} = \boxed{\frac{-16}{105}}$

س ۱۰)

$\underbrace{2\sin\alpha < \sin2\alpha}_{I}$ و $\underbrace{0 < \frac{\cot\alpha}{\sin\alpha}}_{II}$

I: $0 < 2\sin\alpha\cos\alpha - 2\sin\alpha \Rightarrow 0 < 2\sin\alpha(\cos\alpha - 1) \Rightarrow \sin\alpha < 0$

اگر $\cos\alpha \neq 1$ ، $(\cos\alpha - 1)$ همواره نامثبت

II: $\frac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha} > 0 \rightarrow \frac{\cos\alpha}{\sin^2\alpha} > 0$ پس $\cos > 0$

($\sin^2\alpha$ همواره مثبت)

I و II $\Rightarrow$ ربع چهارم