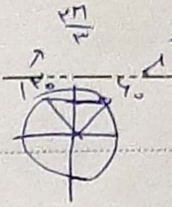
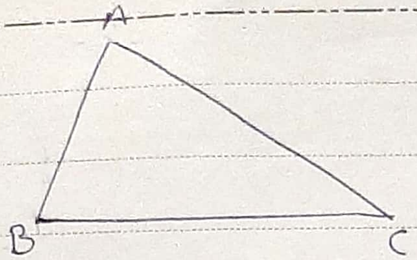
عازم دهم   فاطمه ثابت - ۳۷

۱۸,۵



1 -



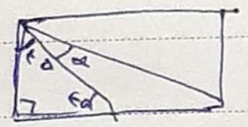
$\frac{1}{2}\times\sqrt{3}\times 6\times\sin\alpha = 4.5$

$\Rightarrow \sin\alpha = \frac{\sqrt{3}}{2} \Rightarrow \alpha = \frac{2\pi}{3}, \frac{\pi}{3}$

$\Rightarrow \frac{\frac{2\pi}{3}}{\frac{\pi}{3}} = 2$

۲

2 -

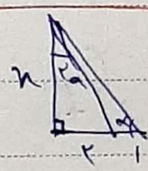


$\cot(45+\alpha) = \frac{\cot 45\times\cot\alpha - 1}{\cot\alpha + \cot 45} = \frac{\cot\alpha - 1}{\cot\alpha + 1} = \frac{1}{2}$

۲

$\Rightarrow \cot\alpha + 1 = 2\cot\alpha - 2 \Rightarrow \cot\alpha = 3$

3 - $\tan 2\alpha = \frac{2}{x}$ $\tan\alpha = \frac{x}{3}$

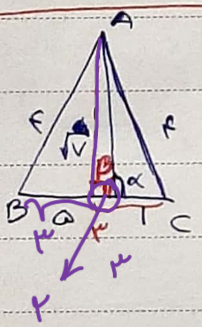


$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{\frac{2x}{3}}{1-\frac{x^2}{9}} = \frac{\frac{2x}{3}}{\frac{9-x^2}{9}} = \frac{6x}{9-x^2} = \frac{2}{x} \Rightarrow 3x^2 = 9 - x^2$

۲

$\Rightarrow 4x^2 = 9 \Rightarrow x = \frac{3}{2} \Rightarrow \cot\alpha = \frac{1}{\tan\alpha} = \frac{3}{x} = \frac{3}{\frac{3}{2}} = 2$

4 -



$\rightarrow \tan\beta = \frac{\sqrt{7}}{3}$ $\rightarrow$ $\alpha+\beta = 180 \Rightarrow \tan\alpha = -\tan\beta$

$\Rightarrow \tan\alpha = -\frac{\sqrt{7}}{3}$ ۲

۱,۵

5 - $2\sin^2 x + \cos^2 x = \frac{4}{3} \Rightarrow \sin^2 x + 1 = \frac{4}{3} \Rightarrow \sin^2 x = \frac{1}{3} \Rightarrow \cos^2 x = \frac{2}{3}$

$\Rightarrow \tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$

۲

6 -

$\cos^2\alpha = 1-\sin^2\alpha$
$\sin^2\alpha = 1-\cos^2\alpha$
$\Rightarrow \frac{\sin^4\alpha + 4 - 4\sin^2\alpha}{2-\sin^2\alpha} - \frac{\cos^4\alpha + 4 - 4\cos^2\alpha}{2-\cos^2\alpha} = \frac{(\sin^2\alpha-2)^2}{-(\sin^2\alpha-2)} - \frac{(\cos^2\alpha-}{-(\cos^2\alpha-}$

۲

$-\sin^2\alpha + 2 - (-\cos^2\alpha + 2) = \cos^2\alpha - \sin^2\alpha = \cos 2\alpha$

۷- $\tan\alpha = \frac{۴}{۳} \Rightarrow \cos\alpha = \sqrt{\frac{۱}{۱+\frac{۱۶}{۹}}} = \frac{۳}{۵} \Rightarrow \sin\alpha = \frac{۴}{۵}$

$\sin(\underbrace{\frac{۹\pi}{۲}}_{۴\pi+\frac{\pi}{۲}}+\alpha)\cos(\underbrace{\frac{۷\pi}{۲}}_{۲\pi+\frac{۳\pi}{۲}}-\alpha) - \underbrace{\tan(\alpha-\frac{۳\pi}{۲})}_{\cot\alpha}$ ۵

$\Rightarrow \sin(\frac{\pi}{۲}+\alpha)\cos(\frac{۳\pi}{۲}-\alpha) - \tan(\alpha-\frac{۳\pi}{۲}) = \cos\alpha \times (-\sin\alpha) + \cot\alpha$

$\Rightarrow \frac{۳}{۵}\times(-\frac{۴}{۵}) + \frac{۳}{۴} = -\frac{۱۲}{۲۵} + \frac{۳}{۴} = \frac{۷۵-۴۸}{۱۰۰} = \boxed{\frac{۲۷}{۱۰۰}}$

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۸ $\;۳\cos\frac{\pi}{۳} + \sqrt{۲}\sin\frac{\pi}{۱۲} - \sqrt{۲}\cos\frac{\pi}{۱۲} = \frac{۳}{۲} + \sqrt{(\sqrt{۲}\sin\frac{\pi}{۱۲} - \sqrt{۲}\cos\frac{\pi}{۱۲})^۲}$

$\frac{۳}{۲} + \sqrt{۲\sin^۲\frac{\pi}{۱۲} + ۲\cos^۲\frac{\pi}{۱۲} - ۴\sin\frac{\pi}{۱۲}\cos\frac{\pi}{۱۲}} = \frac{۳}{۲} + \sqrt{۲ - ۲(\sin\frac{\pi}{۶})}$ ۱

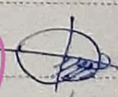
$= \frac{۳}{۲} + \sqrt{۲-۱} = \frac{۳}{۲} + ۱ = \boxed{\frac{۵}{۲}}$

۹- $\frac{\sin\alpha}{۱+\cos\alpha} = \frac{۱-\cos\alpha}{\sin\alpha} = \tan\frac{\alpha}{۲} = \frac{۱}{۴} \Rightarrow$
- $۴\sin\alpha = ۱+\cos\alpha$
- $(۴ - ۴\cos\alpha = \sin\alpha) \times ۴ \to ۱۶ - ۱۶\cos\alpha = ۴\sin\alpha$

$\Rightarrow ۱+\cos\alpha = ۱۶ - ۱۶\cos\alpha \Rightarrow \cos\alpha = \frac{۱۵}{۱۷} \Rightarrow \sin\alpha = \frac{۱+\frac{۱۵}{۱۷}}{۴} = \frac{۸}{۱۷} \Rightarrow \tan\alpha = \frac{۸}{۱۵}$ ۵

$\frac{\frac{۸}{۱۵} - \frac{۸}{۱۷}}{\frac{۸}{۱۷} - \frac{۱۵}{۱۷}} = \frac{\frac{۱۶}{\cancel{۲۵۵}\,۱۵}}{\frac{-۷}{\cancel{۱۷}}} = \boxed{-\frac{۱۶}{۱۰۵}}$

۱۰- $\;۰ < \frac{\cot\alpha}{\sin\alpha} \to ۰ < \frac{\cos\alpha}{\sin^۲\alpha} \Rightarrow \cos\alpha > ۰$
$+ \quad \sin\alpha \neq ۰ \Rightarrow \cos\alpha \neq ۱$ ۵



$۲\sin\alpha < \sin ۲\alpha \Rightarrow ۲\sin\alpha < ۲\sin\alpha\cos\alpha \to ۰ < \sin\alpha(\cos\alpha - ۱)$

$۱ > \cos\alpha > ۰ \Rightarrow \cos\alpha - ۱ < ۰ \Rightarrow \sin\alpha < ۰ \Longrightarrow$ α ناحیه ۴ است

۱) $\frac{۳}{۲} + \sqrt{۲}(\underbrace{\sin\frac{\pi}{۱۲} + \cos\frac{\pi}{۱۲}}_{A})$ $A^۲ = ۱ - \sin\frac{\pi}{۶} = ۱ - \frac{۱}{۲} = \frac{۱}{۲} \xrightarrow[A<۰]{\sqrt{}} A = \frac{-۱}{\sqrt{۲}}$

$\frac{۳}{۲} + \sqrt{۲} \times \frac{-۱}{\sqrt{۲}} = \frac{۱}{۲}$