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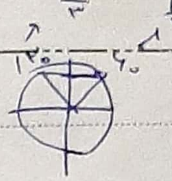
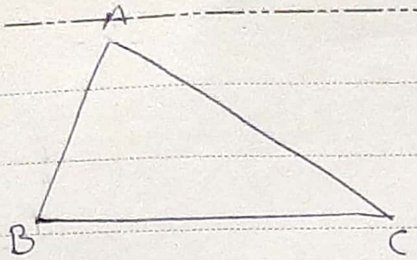


1-

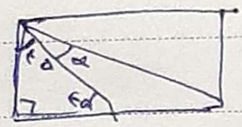


$\frac{1}{2}\times\sqrt{3}\times 6\times\sin\alpha = 4,5$

$\Rightarrow \sin\alpha = \frac{\sqrt{3}}{2} \Rightarrow \alpha = \frac{2\pi}{3} , \frac{\pi}{3}$

$\Rightarrow \frac{\frac{2\pi}{3}}{\frac{\pi}{3}} = \boxed{2}$

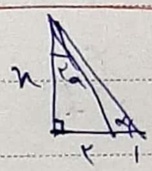
2-



$\cot(45+\alpha) = \frac{\cot 45\times\cot\alpha - 1}{\cot\alpha + \cot 45} = \frac{\cot\alpha - 1}{\cot\alpha + 1} = \frac{1}{2}$

$\Rightarrow \cot\alpha + 1 = 2\cot\alpha - 2 \Rightarrow \cot\alpha = \boxed{3}$

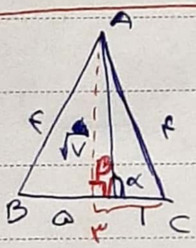
3- $\tan 2\alpha = \frac{2}{x}$ $\tan\alpha = \frac{x}{3}$



$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{\frac{2x}{3}}{1-\frac{x^2}{9}} = \frac{\frac{2x}{3}}{\frac{9-x^2}{9}} = \frac{6x}{9-x^2} = \frac{2}{x} \Rightarrow 3x^2 = 9 - x^2$

$\Rightarrow 4x^2 = 9 \Rightarrow x = \frac{3}{2} \Rightarrow \cot\alpha = \frac{1}{\tan\alpha} = \frac{3}{x} = \frac{3}{\frac{3}{2}} = \boxed{2}$

4-



$\rightarrow \tan\beta = \frac{\sqrt{7}}{3}$ $\rightarrow$ ~~tan~~ $\alpha+\beta = 180 \Rightarrow \tan\alpha = -\tan\beta$

$\Rightarrow \tan\alpha = -\frac{\sqrt{7}}{3}$

5- $2\sin^2 x + \cos^2 x = \frac{4}{3} \Rightarrow \sin^2 x + 1 = \frac{4}{3} \Rightarrow \sin^2 x = \frac{1}{3} \Rightarrow \cos^2 x = \frac{2}{3}$

$\Rightarrow \tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$

6- [illegible]

$\cos^2\alpha = 1-\sin^2\alpha$
$\sin^2\alpha = 1-\cos^2\alpha$ $\Rightarrow \frac{\sin^4\alpha + 4 - 4\sin^2\alpha}{2-\sin^2\alpha} - \frac{\cos^4\alpha + 4 - 4\cos^2\alpha}{2-\cos^2\alpha} = \frac{(\sin^2\alpha - 2)^2}{-(\sin^2\alpha - 2)} - \frac{(\cos^2\alpha - }{-(\cos^2\alpha - }$

$-\sin^2\alpha + 2 - (-\cos^2\alpha + 2) = \cos^2\alpha - \sin^2\alpha = \boxed{\cos 2\alpha}$

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7- $\tan\alpha = \frac{4}{3} \Rightarrow \cos\alpha = \sqrt{\frac{1}{1+\frac{16}{9}}} = \frac{3}{5} \Rightarrow \sin\alpha = \frac{4}{5}$

$\sin\Big(\underbrace{\frac{9\pi}{2}}_{4\pi+\frac{\pi}{2}}+\alpha\Big)\cos\Big(\underbrace{\frac{7\pi}{2}}_{2\pi+\frac{3\pi}{2}}-\alpha\Big) - \underbrace{\tan\Big(\alpha-\frac{3\pi}{2}\Big)}_{\cot\alpha}$

$\Rightarrow \sin\left(\frac{\pi}{2}+\alpha\right)\cos\left(\frac{3\pi}{2}-\alpha\right)-\tan\left(\alpha-\frac{3\pi}{2}\right) = \cos\alpha\times(-\sin\alpha)+\cot\alpha$

$\Rightarrow \frac{3}{5}\times\left(-\frac{4}{5}\right)+\frac{3}{4} = -\frac{12}{25}+\frac{3}{4} = \frac{75-48}{100} = \boxed{\frac{27}{100}}$

8- $3\cos\frac{\pi}{3} + \sqrt{2}\sin\frac{\pi}{12} - \sqrt{2}\cos\frac{\pi}{12} = \frac{3}{2} + \sqrt{\left(\sqrt{2}\sin\frac{\pi}{12}-\sqrt{2}\cos\frac{\pi}{12}\right)^2}$

$\frac{3}{2}+\sqrt{2\sin^2\frac{\pi}{12}+2\cos^2\frac{\pi}{12}-4\sin\frac{\pi}{12}\cos\frac{\pi}{12}} = \frac{3}{2}+\sqrt{2-2\left(\sin\frac{\pi}{6}\right)}$

$= \frac{3}{2}+\sqrt{2-1} = \frac{3}{2}+1 = \boxed{\frac{5}{2}}$

9- $\frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha} = \tan\frac{\alpha}{2} = \frac{1}{4} \Rightarrow$

$4\sin\alpha = 1+\cos\alpha$

$(4-4\cos\alpha = \sin\alpha)\times 4 \rightarrow 16-16\cos\alpha = 4\sin\alpha$

$\Rightarrow 1+\cos\alpha = 16-16\cos\alpha \Rightarrow \cos\alpha = \frac{15}{17} \Rightarrow \sin\alpha = \frac{1+\frac{15}{17}}{4} = \frac{8}{17} \Rightarrow \tan\alpha = \frac{8}{15}$

$\rightarrow \frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}} = \frac{\frac{16}{255}}{\frac{-7}{17}} = \boxed{-\frac{16}{105}}$

10- $0<\frac{\cot\alpha}{\sin\alpha} \rightarrow 0<\frac{\cos\alpha}{\sin^2\alpha} \Rightarrow \cos\alpha>0$

$+\ \sin\alpha\neq 0 \Rightarrow \cos\alpha\neq 1$

$2\sin\alpha<\sin 2\alpha \Rightarrow 2\sin\alpha<2\sin\alpha\cos\alpha \rightarrow 0<\sin\alpha(\cos\alpha-1)$

$1>\cos\alpha>0 \Rightarrow \cos\alpha-1<0 \Rightarrow \sin\alpha<0 \Longrightarrow$ α ناحیه ۴ ام