تکلیف ۲۷ - یازدهم ریاضی - برنامه طاهریان

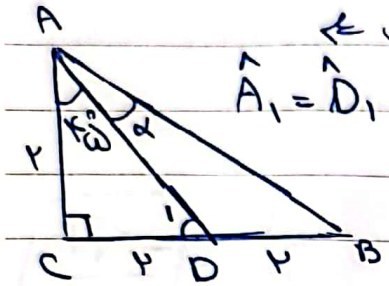
۱-

$$S = \frac{1}{2} \times a \times b \times \sin\alpha \Rightarrow \frac{1}{2} \times \sqrt{3} \times 4 \times \sin\alpha = \frac{9}{2}\sqrt{3}$$

$$\Rightarrow \sin\alpha = \frac{\sqrt{3}}{2} \rightarrow \alpha = \frac{\pi}{3} \quad \text{یا} \quad \alpha = \frac{2\pi}{3}$$

$$\frac{\frac{2\pi}{3}}{\frac{\pi}{3}} = \boxed{2}$$

۲- مثلث ACD قائم‌الزاویه متساوی‌الساقین است ← $\hat{A}_1 = \hat{D}_1 = 45^\circ$



$$AB = \sqrt{4+14} = 2\sqrt{6}$$

$$\sin(\alpha + 45^\circ) = \sin\alpha\cos 45^\circ + \cos\alpha \sin 45^\circ$$

$$= \frac{\sqrt{2}}{2}\sin\alpha + \frac{\sqrt{2}}{2}\cos\alpha = \frac{4}{2\sqrt{6}} = \frac{2}{\sqrt{6}}$$

$$\Rightarrow \frac{\sqrt{2}}{2}(\sin\alpha + \cos\alpha) = \frac{2}{\sqrt{6}}$$

$$\times\sqrt{2} \rightarrow \sin\alpha + \cos\alpha = \frac{2\sqrt{2}}{\sqrt{6}} \quad (1)$$

$$\cos(\alpha + 45^\circ) = \cos\alpha\cos 45^\circ - \sin\alpha\sin 45^\circ =$$

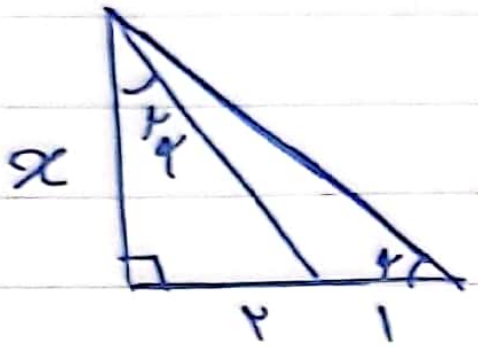
$$\frac{\sqrt{2}}{2}\cos\alpha - \frac{\sqrt{2}}{2}\sin\alpha = \frac{2}{2\sqrt{6}} = \frac{1}{\sqrt{6}} \Rightarrow \cos\alpha - \sin\alpha = \frac{\sqrt{2}}{\sqrt{6}} \quad (2)$$

$$(1)+(2) \rightarrow 2\cos\alpha = \frac{3\sqrt{2}}{\sqrt{6}} \Rightarrow \cos\alpha = \frac{1/5\sqrt{2}}{\sqrt{6}}$$

$$\Rightarrow \sin\alpha = \frac{0/5\sqrt{2}}{\sqrt{6}}$$

$$\Rightarrow \cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \boxed{3}$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$



من الشكل المقابل : $\tan 2\alpha = \frac{2}{x} \Rightarrow$

$$\tan\alpha = \frac{x}{3}$$

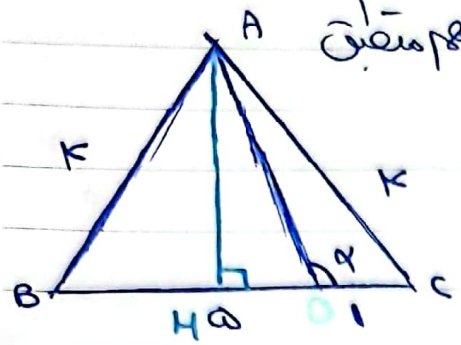
$$\frac{2}{x} = \frac{\frac{2}{3}x}{1-\frac{x^2}{9}} = \frac{\frac{2x}{3}}{\frac{9-x^2}{9}} \Rightarrow \frac{3x}{9-x^2} = \frac{1}{x}$$

$$\Rightarrow 3x^2 = 9 - x^2 \Rightarrow 4x^2 = 9$$

$$x^2 = \frac{9}{4} \Rightarrow x = \frac{3}{2}$$

$$\cot\alpha = \frac{3}{x} = \frac{3}{\frac{3}{2}} = \boxed{2}$$

۴- مثلث متساوی الساقین است ⇐ ارتفاع وارد بر قاعده متوسط



هستند ⇐ $BH = CH = ۳$

$\Rightarrow (AH)^۲ + (CH)^۲ = (AC)^۲$

$(AH)^۲ + ۹ = ۱۶$

$\Rightarrow (AH)^۲ = ۷ \Rightarrow AH = \sqrt{۷}$

مثلث AHD: $HD = ۲$ ، $AH = \sqrt{۷}$

$\Rightarrow \cot(۱۸۰ - \alpha) = \frac{HD}{AH} = \frac{۲}{\sqrt{۷}}$

$\hookrightarrow -\cot\alpha \qquad \Rightarrow \cot\alpha = -\frac{۲}{\sqrt{۷}}$

$\Rightarrow \tan\alpha = -\frac{\sqrt{۷}}{۲}$

۵- $۲\sin^۲x + \cos^۲x = \sin^۲x + \underbrace{\cos^۲x + \sin^۲x}_{۱} = ۱ + \sin^۲x = \frac{۴}{۳}$

$\Rightarrow \sin^۲x = \frac{۱}{۳} \Rightarrow \cos^۲x = \frac{۲}{۳} \qquad \tan^۲x = \frac{\sin^۲x}{\cos^۲x} = \frac{۱}{۲}$

۶- $\frac{\sin^۴\alpha + ۴\cos^۲\alpha}{۱ + \cos^۲\alpha} = \frac{\sin^۴\alpha - ۴\sin^۲\alpha + ۴}{۲ - \sin^۲\alpha} = \frac{(۲ - \sin^۲\alpha)^۲}{۲ - \sin^۲\alpha}$

$\cos^۲\alpha = ۱ - \sin^۲\alpha \qquad = ۲ - \sin^۲\alpha$

$\frac{\cos^۴\alpha + ۴\sin^۲\alpha}{۱ + \sin^۲\alpha} = \frac{\cos^۴\alpha - ۴\cos^۲\alpha + ۴}{۲ - \cos^۲\alpha} = \frac{(۲ - \cos^۲\alpha)^۲}{۲ - \cos^۲\alpha}$

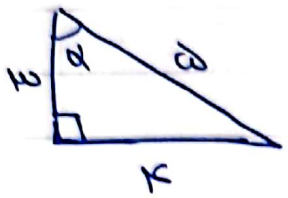
$\sin^۲\alpha = ۱ - \cos^۲\alpha \qquad = ۲ - \cos^۲\alpha$

$$2 - \sin^2\alpha - 2 + \cos^2\alpha = \cos^2\alpha - \sin^2\alpha = \boxed{\cos 2\alpha}$$

$$\sin\left(\frac{9\pi}{2} + \alpha\right)\cos\left(\frac{7\pi}{2} - \alpha\right) - \tan\left(\alpha - \frac{3\pi}{2}\right) = -1$$

$$\sin\left(\frac{\pi}{2} + \alpha\right)\cos\left(\frac{3\pi}{2} - \alpha\right) - \tan\left(\alpha - \frac{3\pi}{2}\right) =$$

$$(+\cos\alpha)\times(-\sin\alpha) - (-\cot\alpha) = \cot\alpha - \sin\alpha\cos\alpha$$



$$\tan\alpha = \frac{4}{3} \quad \cot\alpha = \frac{3}{4} \qquad \frac{3}{4} - \left(\frac{-3}{5}\right)\times\left(\frac{-4}{5}\right)$$

$$\sin\alpha = \frac{-4}{5} \quad \cos\alpha = \frac{-3}{5} \qquad = \frac{3}{4} - \frac{12}{20} =$$

$$0{,}75 - 0{,}48 = \boxed{0{,}27}$$

$$\sin 2x = 2\sin x\cos x \Rightarrow \sin\frac{\pi}{6} = 2\sin\frac{\pi}{12}\cos\frac{\pi}{12} = \frac{1}{2}$$

$$\Rightarrow \sin\frac{\pi}{12}\cos\frac{\pi}{12} = \frac{1}{4}$$

$$\cos 2x = \cos^2 x - \sin^2 x \Rightarrow \cos\frac{\pi}{6} = \cos^2\frac{\pi}{12} - \sin^2\frac{\pi}{12}$$

$$= \frac{\sqrt{3}}{2} \quad (1)$$

$$\left(\sin\frac{\pi}{12} + \cos\frac{\pi}{12}\right)^2 = \underbrace{\sin^2\frac{\pi}{12} + \cos^2\frac{\pi}{12}}_{1} + 2\sin\frac{\pi}{12}\cos\frac{\pi}{12}$$

$$= 1 + 2\times\frac{1}{4} = \frac{3}{2}$$

$$\Rightarrow \sin\frac{\pi}{12} + \cos\frac{\pi}{12} = \frac{\sqrt{3}}{\sqrt{2}} \quad (2)$$

$$\Rightarrow \cos^2\frac{\pi}{12} - \sin^2\frac{\pi}{12} = \left(\cos\frac{\pi}{12} - \sin\frac{\pi}{12}\right)\left(\cos\frac{\pi}{12} + \sin\frac{\pi}{12}\right)$$

$$= \frac{\sqrt{10}}{\sqrt{5}} \times \left(\cos\frac{\pi}{4} - \sin\frac{\pi}{12}\right) = \frac{\sqrt{10}}{4\sqrt{2}} \Rightarrow$$

$$\cos\frac{\pi}{12} - \sin\frac{\pi}{12} = \frac{1}{\sqrt{2}} \Rightarrow \sqrt{2}\sin x - \sqrt{2}\cos x = -1$$

$$3\cos\frac{\pi}{3} - 1 = \frac{3}{2} - 1 = \boxed{\frac{1}{2}}$$

$$\sin x - \cos x = \sqrt{2}\sin\left(x - \frac{\pi}{4}\right) \quad -9$$

$$\times\sqrt{2} \rightarrow \sqrt{2}(\sin x - \cos x) = 2\sin\left(x - \frac{\pi}{4}\right) = 2\sin\left(\frac{\pi}{12} - \frac{\pi}{4}\right) \quad (9)$$

$$= 2\sin\left(-\frac{\pi}{6}\right) = -1$$

$$3\cos\left(2 \times \frac{\pi}{12}\right) = 3\cos\frac{\pi}{6} = \frac{3}{2}$$

$$\frac{3}{2} - 1 = \boxed{\frac{1}{2}}$$

$$\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1 - \tan^2\frac{\alpha}{2}} = \frac{8}{15} \qquad \cos\alpha = \frac{15}{17},\ \sin\alpha = \frac{8}{17}$$

$$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{-14}{105}$$

$$2\sin\alpha < \sin 2\alpha \Rightarrow 2\sin\alpha < 2\sin\alpha\cos\alpha \quad -10$$

$\sin\alpha > 0 \rightarrow \cos\alpha < 1$ و $\sin\alpha < 0 \rightarrow \cos\alpha > 1$ و

$$\Rightarrow \sin\alpha < 0 \qquad \frac{\cot\alpha}{\sin\alpha} > 0 \Rightarrow \frac{\cos\alpha}{\sin^2\alpha} > 0 \quad (6)$$

$\Rightarrow \cos\alpha > 0$ انتهای کمان α در ربع ۴ قرار دارد