ترانه طاهریان - تکلیف ۲۷ - یازدهم ریاضی

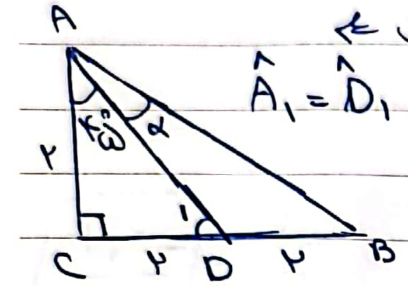
۱-

$$S = \frac{1}{2} \times a \times b \times \sin\alpha \Rightarrow \frac{1}{\cancel{2}} \times \cancel{\sqrt{3}} \times \cancel{9} \times \sin\alpha = \frac{9}{\cancel{2}} \sqrt{3}$$

$$\Rightarrow \sin\alpha = \frac{\sqrt{3}}{2} \rightarrow \alpha = \frac{\pi}{3} \qquad \alpha = \frac{2\pi}{3} \qquad \frac{\frac{2\pi}{3}}{\frac{\pi}{3}} = \boxed{2}$$

۲- مثلث $A\hat{C}D$ قائم الزاویه‌ی متساوی الساقین است ⇐

$$\hat{A}_1 = \hat{D}_1 = 45^\circ$$



$$\sin(\alpha + 45^\circ) = \sin\alpha\cos45^\circ + \cos\alpha\sin45^\circ$$

$$= \frac{\sqrt{2}}{2}\sin\alpha + \frac{\sqrt{2}}{2}\cos\alpha = \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}}$$

$$AB = \sqrt{4+16} = 2\sqrt{5}$$

$$\Rightarrow \frac{\sqrt{2}}{2}(\sin\alpha + \cos\alpha) = \frac{2}{\sqrt{5}}$$

$$\xrightarrow{\times\sqrt{2}} \sin\alpha + \cos\alpha = \frac{2\sqrt{2}}{\sqrt{5}} \quad (1)$$

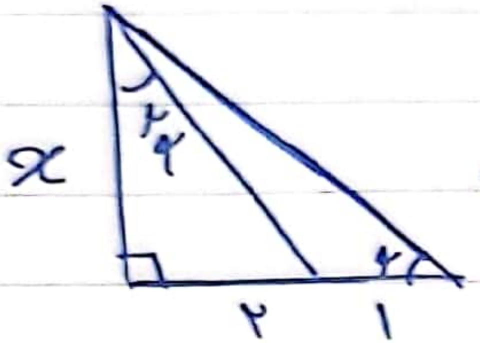
$$\cos(\alpha + 45^\circ) = \cos\alpha\cos45^\circ - \sin\alpha\sin45^\circ =$$

$$\frac{\sqrt{2}}{2}\cos\alpha - \frac{\sqrt{2}}{2}\sin\alpha = \frac{2}{2\sqrt{5}} = \frac{1}{\sqrt{5}} \Rightarrow \cos\alpha - \sin\alpha = \frac{\sqrt{2}}{\sqrt{5}} \quad (2)$$

$$\xrightarrow{(1)(2)} 2\cos\alpha = \frac{3\sqrt{2}}{\sqrt{5}} \Rightarrow \cos\alpha = \frac{1.5\sqrt{2}}{\sqrt{5}}$$

$$\Rightarrow \sin\alpha = \frac{0.5\sqrt{2}}{\sqrt{5}}$$

$$\Rightarrow \cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \boxed{3}$$



$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

طبق شکل : $\tan 2\alpha = \frac{2}{x} \Rightarrow$

$\tan\alpha = \frac{x}{3}$

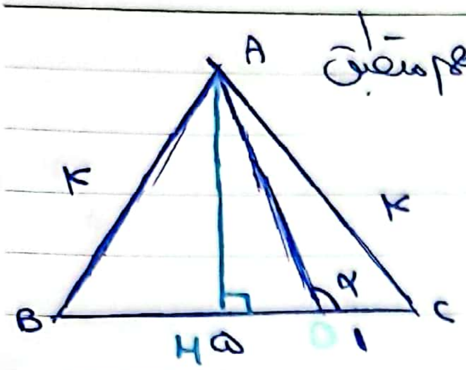
$$\frac{\cancel{2}}{x} = \frac{\frac{2}{3}x}{1-\frac{x^2}{9}} = \frac{\frac{\cancel{2}x}{3}}{\frac{9-x^2}{9}} \Rightarrow \frac{3x}{9-x^2} = \frac{1}{x}$$

$$\Rightarrow 3x^2 = 9 - x^2 \Rightarrow 4x^2 = 9$$

$$x^2 = \frac{9}{4} \Rightarrow x = \frac{3}{2}$$

$$\cot\alpha = \frac{3}{x} = \frac{3}{\frac{3}{2}} = \boxed{2}$$

۴- مثلث مساوی الساقین است ⇐ ارتفاع و میانه بر هم منطبق هستند ⇐ $BH = CH = 3$



$$\Rightarrow (AH)^2 + (CH)^2 = (AC)^2$$
$$(AH)^2 + 9 = 16$$
$$\Rightarrow (AH)^2 = 7 \Rightarrow AH = \sqrt{7}$$

در مثلث $\overset{\triangle}{AHD}$: $HD = 2$ ، $AH = \sqrt{7}$

$$\Rightarrow \cot(180 - \alpha) = \frac{HD}{AH} = \frac{2}{\sqrt{7}}$$

$\hookrightarrow -\cot\alpha \quad \Rightarrow \cot\alpha = \frac{-2}{\sqrt{7}}$

$$\Rightarrow \tan\alpha = \frac{-\sqrt{7}}{2}$$

۵-

$$2\sin^2 x + \cos^2 x = \sin^2 x + \underbrace{\cos^2 x + \sin^2 x}_{1} = 1 + \sin^2 x = \frac{4}{3}$$

$$\Rightarrow \sin^2 x = \frac{1}{3} \Rightarrow \cos^2 x = \frac{2}{3} \qquad \tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \boxed{\frac{1}{2}}$$

۶-

$$\frac{\sin^4\alpha + 4\cos^2\alpha}{1 + \cos^2\alpha} = \frac{\sin^4\alpha - 4\sin^2\alpha + 4}{2 - \sin^2\alpha} = \frac{(2 - \sin^2\alpha)^2}{2 - \sin^2\alpha}$$

$\cos^2\alpha = 1 - \sin^2\alpha$

$$= 2 - \sin^2\alpha$$

$$\frac{\cos^4\alpha + 4\sin^2\alpha}{1 + \sin^2\alpha} = \frac{\cos^4\alpha - 4\cos^2\alpha + 4}{2 - \cos^2\alpha} = \frac{(2 - \cos^2\alpha)^2}{2 - \cos^2\alpha}$$

$\sin^2\alpha = 1 - \cos^2\alpha$

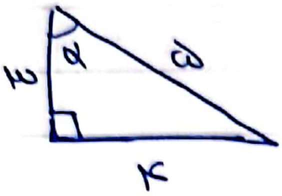
$$= 2 - \cos^2\alpha$$

$$۲ - \sin^۲\alpha - ۲ + \cos^۲\alpha = \cos^۲\alpha - \sin^۲\alpha = \boxed{\cos ۲\alpha}$$

$$\sin\left(\frac{۹\pi}{۲} + \alpha\right)\cos\left(\frac{۷\pi}{۲} - \alpha\right) - \tan\left(\alpha - \frac{۳\pi}{۲}\right) = \qquad -۷$$

$$\sin\left(\frac{\pi}{۲} + \alpha\right)\cos\left(\frac{۳\pi}{۲} - \alpha\right) - \tan\left(\alpha - \frac{۳\pi}{۲}\right) =$$

$$(+\cos\alpha)\times(-\sin\alpha) - (-\cot\alpha) = \cot\alpha - \sin\alpha\cos\alpha$$



$$\tan\alpha = \frac{۴}{۳} \quad \cot\alpha = \frac{۳}{۴} \quad \frac{۳}{۴} - \left(\frac{-۳}{۵}\right)\times\left(\frac{-۴}{۵}\right)$$

$$\sin\alpha = \frac{-۴}{۵} \quad \cos\alpha = \frac{-۳}{۵} \quad = \frac{۳}{۴} - \frac{۱۲}{۲۵} =$$

$$۰٫۷۵ - ۰٫۴۸ = \boxed{۰٫۲۷}$$

$$\sin ۲x = ۲\sin x\cos x \Rightarrow \sin\frac{\pi}{۶} = ۲\sin\frac{\pi}{۱۲}\cos\frac{\pi}{۱۲} = \frac{۱}{۲} \qquad -۸$$

$$\Rightarrow \sin\frac{\pi}{۱۲}\cos\frac{\pi}{۱۲} = \frac{۱}{۴}$$

$$\cos ۲x = \cos^۲ x - \sin^۲ x \Rightarrow \cos\frac{\pi}{۶} = \cos^۲\frac{\pi}{۱۲} - \sin^۲\frac{\pi}{۱۲}$$

$$= \frac{\sqrt{۳}}{۲} \quad (۱)$$

$$\left(\sin\frac{\pi}{۱۲} + \cos\frac{\pi}{۱۲}\right)^۲ = \underbrace{\sin^۲\frac{\pi}{۱۲} + \cos^۲\frac{\pi}{۱۲}}_{۱} + ۲\sin\frac{\pi}{۱۲}\cos\frac{\pi}{۱۲}$$

$$= ۱ + ۲\times\frac{۱}{۴} = \frac{۳}{۲}$$

$$\Rightarrow \sin\frac{\pi}{۱۲} + \cos\frac{\pi}{۱۲} = \frac{\sqrt{۳}}{\sqrt{۲}} \quad (۲)$$

$$\Rightarrow \cos^۲\frac{\pi}{۱۲} - \sin^۲\frac{\pi}{۱۲} = \left(\cos\frac{\pi}{۱۲} - \sin\frac{\pi}{۱۲}\right)\left(\cos\frac{\pi}{۱۲} + \sin\right.$$

/ /

$= \dfrac{\cancel{\sqrt{۱۵}}}{\cancel{\sqrt{۶}}} \times \left(\cos\dfrac{\pi}{۱۲} - \sin\dfrac{\pi}{۱۲}\right) = \dfrac{\cancel{\sqrt{۱۵}}}{\dfrac{\cancel{۳}}{\sqrt{۲}}} \Rightarrow$

$\cos\dfrac{\pi}{۱۲} - \sin\dfrac{\pi}{۱۲} = \dfrac{۱}{\sqrt{۲}} \Rightarrow \sqrt{۲}\sin x - \sqrt{۲}\cos x = -۱$

$۳\cos\dfrac{\pi}{۳} - ۱ = \dfrac{۳}{۲} - ۱ = \boxed{\dfrac{۱}{۲}}$

$\sin x - \cos x = \sqrt{۲}\sin\left(x - \dfrac{\pi}{۴}\right)$ ۹-

$\xrightarrow{\times\sqrt{۲}} \sqrt{۲}(\sin x - \cos x) = ۲\sin\left(x - \dfrac{\pi}{۴}\right) = ۲\sin\left(\dfrac{\pi}{۱۲} - \dfrac{\pi}{۴}\right)$

$= ۲\sin\left(-\dfrac{\pi}{۶}\right) = -۱$

$۳\cos\left(۴\times\dfrac{\pi}{۱۲}\right) = ۳\cos\dfrac{\pi}{۳} = \dfrac{۳}{۲}$

$\dfrac{۳}{۲} - ۱ = \boxed{\dfrac{۱}{۲}}$

$۲\sin\alpha < \sin ۲\alpha \Rightarrow ۲\sin\alpha < ۲\sin\alpha\cos\alpha$ ۱۰-

$\xrightarrow{\sin\alpha > ۰} \cos\alpha > ۱$ غ ق ق $\xrightarrow{\sin\alpha < ۰} \cos\alpha < ۱$ ق ق

$\Rightarrow \sin\alpha < ۰$ $\qquad \dfrac{\cot\alpha}{\sin\alpha} > ۰ \Rightarrow \dfrac{\cos\alpha}{\sin^۲\alpha} > ۰$

$\Rightarrow \cos\alpha > ۰$ انتهای کمان α در ربع ۴ قرار دارد