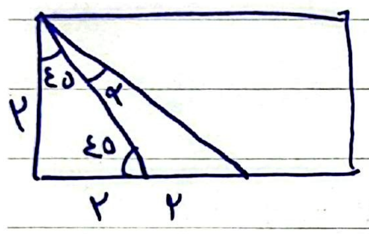
(حسابان مختاری تکلیف ۲۷) ۲۰

① $S = \frac{1}{2}\times\sqrt{3}\times 6\times \sin\alpha = 4.5$

$\sin\alpha = \frac{4.5}{3\sqrt{3}} = \frac{\sqrt{3}}{2}$ $\alpha = 60^\circ,\ 120^\circ$ $\frac{120}{60} = ②$ ۴

②



$\cot(\alpha + 45) = \frac{1}{2}$ ۴

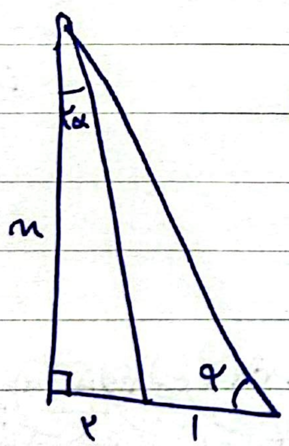
$\cot(\alpha+45) = \frac{1-\tan\alpha\tan 45}{\tan\alpha + \tan 45} = \frac{1-\tan\alpha}{\tan\alpha + 1}$

$\frac{1-\tan\alpha}{1+\tan\alpha} = \frac{1}{2} \rightarrow 2 - 2\tan\alpha = 1 + \tan\alpha$

$\tan\alpha = 1$ $\tan\alpha = \frac{1}{3}$

$\boxed{\cot\alpha = 3}$

③



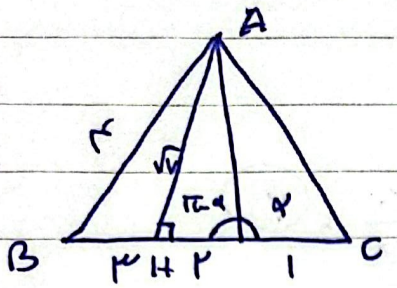
$\tan\alpha = \frac{m}{3}$ ۵

$\tan 2\alpha = \frac{2}{m} \rightarrow \tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{\frac{2m}{3}}{\frac{9-m^2}{9}} = \frac{6m}{9-m^2}$

$\frac{6m}{9-m^2} = \frac{2}{m} \rightarrow 6m^2 = 18 - 2m^2 \rightarrow 8m^2 = 18$

$m^2 = \frac{9}{4} \rightarrow m = \pm\frac{3}{2}$ غ ق ق

$\tan\alpha = \frac{\frac{3}{2}}{3} = \frac{1}{2} \Rightarrow \boxed{\cot\alpha = 2}$



۴)

$\overset{\Delta}{AHC} \Rightarrow AH^2 + 9 = 14 \rightarrow AH = \sqrt{5}$

$\overset{\Delta}{AEH} \Rightarrow \tan(180 - \alpha) = \frac{\sqrt{5}}{2}$

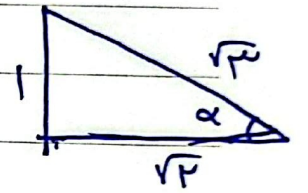
$\tan(180 - \alpha) = -\tan\alpha = \boxed{-\frac{\sqrt{5}}{2}}$

۵)

$\sin^2 x + \underbrace{\sin^2 x + \cos^2 x}_{1} = \frac{5}{3}$

$\sin^2 x = \frac{5}{3} - 1 \Rightarrow \frac{1}{3} \quad \sin x = \pm\sqrt{\frac{1}{3}} = \pm\frac{1}{\sqrt{3}}$

$\cos x = \pm\frac{\sqrt{2}}{\sqrt{3}}$



$\tan x = \frac{1}{\sqrt{2}} \rightarrow \tan^2 x = \boxed{\frac{1}{2}}$

۷)

$\sin x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$\sin^2 x = 1 - \cos^2 x$

$\frac{\sin\alpha^4 + 4(1-\sin^2\alpha)}{1+(1-\sin^2\alpha)} - \frac{\cos^4\alpha + 4(1-\cos^2\alpha)}{1+1-\cos^2\alpha} \Rightarrow$

$\frac{\sin^4\alpha + 4 - 4\sin^2\alpha}{2-\sin^2\alpha} - \frac{\cos^4\alpha + 4 - 4\cos^2\alpha}{2-\cos^2\alpha} = \frac{(2-\sin^2\alpha)^2}{2-\sin^2\alpha} - \frac{(2-\cos^2\alpha)^2}{2-\cos^2\alpha}$

$= 2 - \sin^2\alpha - (2 - \cos^2\alpha) = 2 - \sin^2\alpha - 2 + \cos^2\alpha = \cos^2\alpha - \sin^2\alpha = \boxed{\cos 2\alpha}$

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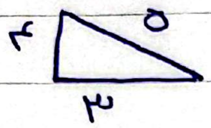


سینوس و کسینوس در ربع ۳ منفی

$\cos = -\frac{3}{5}$

$\sin = -\frac{4}{5}$

۲

$\sin(\frac{9\pi}{2}+\alpha)\cos(\frac{7\pi}{2}-\alpha)-\tan(\alpha-\frac{3\pi}{2})$

$\Rightarrow \cos\alpha \times -\sin\alpha + \cot\alpha$

$(-\frac{3}{5}\times\frac{4}{5}) + \frac{3}{4} \Rightarrow -\frac{48}{100}+\frac{75}{100} \Rightarrow \boxed{\frac{27}{100}}$

۸

$\sin x - \cos x = \sqrt{2}\sin(x-\frac{\pi}{4})$

$\sqrt{2}(\sin x-\cos x) = \sqrt{2}\times\sqrt{2}\times\sin(\frac{\pi}{12}-\frac{\pi}{4}) = 2\sin(\underbrace{-\frac{\pi}{6}}_{-\frac{1}{2}}) = -1$

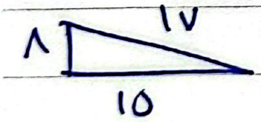
۲

$3\cos(\underbrace{4\times\frac{\pi}{12}}_{\frac{\pi}{3}}) = \frac{3}{2}$

$\frac{3}{2}-1 \Rightarrow \frac{1}{2}$

۹

$\tan\alpha = \frac{2\tan(\frac{\alpha}{2})}{1-\tan^2(\frac{\alpha}{2})} = \frac{2\times\frac{1}{4}}{1-\frac{1}{16}} = \frac{\frac{1}{2}}{\frac{15}{16}} = \frac{16}{30} = \frac{8}{15}$



$\frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha} = \frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}} \Rightarrow \boxed{-\frac{16}{105}}$

۲

۱۰

$\frac{\frac{\cos\alpha}{\sin\alpha}}{\frac{\sin\alpha}{1}} > 0$ $\quad \frac{\cos\alpha}{\sin^2\alpha} > 0 \xrightarrow{\text{پس}} \cos\alpha > 0$

($\sin^2\alpha$: همواره مثبت)

۲

$\sin2\alpha = 2\sin\alpha\cos\alpha \rightarrow 2\sin\alpha < 2\sin\alpha\cos\alpha \Rightarrow$ ($\sin\alpha>0$) $\cos\alpha > 1$ غ.ق.ق

$\sin<0$: $\cos\alpha < 1$ ق.ق → ۴

$\cos +$, $\sin -$ → ناحیه ۴