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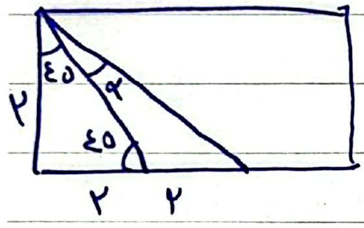
(مهسا مختاری تکلیف ۲۷)

① $S = \frac{1}{2} \times \sqrt{3} \times 6 \times \sin\alpha = 4.5$

② $\sin\alpha = \frac{4.5}{3\sqrt{3}} = \frac{\sqrt{3}}{2}$ $\quad \alpha = 60^\circ,\ 120^\circ$ $\quad \frac{120}{60} = 2$

③



$\cot(\alpha + 45) = \frac{1}{2}$

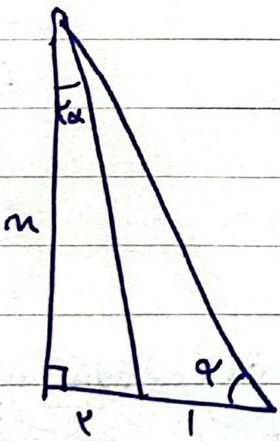
$\cot(\alpha + 45) = \frac{1 - \tan\alpha \tan 45}{\tan\alpha + \tan 45^\circ} = \frac{1 - \tan\alpha}{\tan\alpha + 1}$

$\frac{1 - \tan\alpha}{1 + \tan\alpha} = \frac{1}{2} \to 2 - 2\tan\alpha = 1 + \tan\alpha$

$\tan\alpha = 1 \quad \tan\alpha = \frac{1}{3}$

$\boxed{\cot\alpha = 3}$

③



$\tan\alpha = \frac{m}{3}$

$\tan 2\alpha = \frac{2}{m} \to \tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha} = \frac{\frac{2m}{3}}{\frac{9 - m^2}{9}} = \frac{6m}{9 - m^2}$

$\frac{6m}{9 - m^2} = \frac{2}{m} \to 6m^2 = 18 - 2m^2 \to 8m^2 = 18$

$m^2 = \frac{9}{4} \to m = \pm\frac{3}{2}$ غ ق ق

$\tan\alpha = \frac{\frac{3}{2}}{3} = \frac{1}{2} \Rightarrow \boxed{\cot\alpha = 2}$

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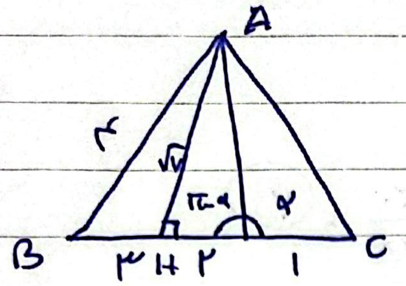

$$\overset{\triangle}{AHC} \Rightarrow AH^2+9=16 \rightarrow AH=\sqrt{7}$$

$$\overset{\triangle}{AEH} \Rightarrow \tan(180-\alpha)=\frac{\sqrt{7}}{2}$$

$$\tan(180-\alpha)=-\tan\alpha=\boxed{-\frac{\sqrt{7}}{2}}$$

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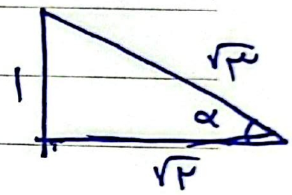
$$\sin^2 x+\underbrace{\sin^2 x+\cos^2 x}_{1}=\frac{4}{3}$$

$$\sin^2 x=\frac{4}{3}-1 \Rightarrow \frac{1}{3} \qquad \sin x=\pm\frac{\sqrt{3}}{3} \text{ یا } \pm\frac{1}{\sqrt{3}}$$

$$\cos x=\pm\frac{\sqrt{2}}{\sqrt{3}}$$



$$\tan x=\frac{1}{\sqrt{2}} \rightarrow \tan^2 x=\frac{1}{2}$$

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$$\sin x+\cos^2 x=1$$

$$\cos^2 x=1-\sin^2 x$$

$$\sin^2 x=1-\cos^2 x$$

$$\frac{\sin^4\alpha+4(1-\sin^2\alpha)}{1+(1-\sin^2\alpha)}-\frac{\cos^4\alpha+4(1-\cos^2\alpha)}{1+1-\cos^2\alpha} \Rightarrow$$

$$\frac{\sin^4\alpha+4-4\sin^2\alpha}{2-\sin^2\alpha}-\frac{\cos^4\alpha+4-4\cos^2\alpha}{2-\cos^2\alpha}=\frac{(2-\sin^2\alpha)^2}{2-\sin^2\alpha}-\frac{(2-\cos^2\alpha)^2}{2-\cos^2\alpha}$$

$$=2-\sin^2\alpha-(2-\cos^2\alpha)=2-\sin^2\alpha-2+\cos^2\alpha=\cos^2\alpha-\sin^2\alpha=\boxed{\cos 2\alpha}$$

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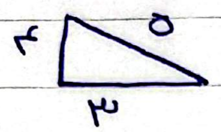

سینوس و کسینوس در ربع ۳ منفی

$\cos = -\frac{3}{5}$

$\sin = -\frac{4}{5}$

$$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right)-\tan\left(\alpha-\frac{3\pi}{2}\right)$$

$$\Rightarrow \cos\alpha \times -\sin\alpha + \cot\alpha$$

$$\left(-\frac{3}{5}\times\frac{4}{5}\right)+\frac{3}{4} \Rightarrow -\frac{48}{100}+\frac{75}{100} \Rightarrow \boxed{\frac{27}{100}}$$

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$$\sin x-\cos x=\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)$$

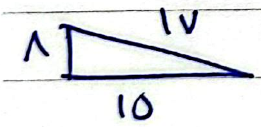
$$\sqrt{2}(\sin x-\cos x)=\sqrt{2}\times\sqrt{2}\times\sin\left(\frac{\pi}{12}-\frac{\pi}{4}\right)=2\underbrace{\sin\left(-\frac{\pi}{6}\right)}_{-\frac{1}{2}}=-1$$

$$3\cos\left(\underbrace{4\times\frac{\pi}{12}}_{\frac{\pi}{3}}\right)=\frac{3}{2}$$

$$\frac{3}{2}-1 \Rightarrow \frac{1}{2}$$

۹

$$\tan\alpha=\frac{2\tan\left(\frac{\alpha}{2}\right)}{1-\tan^2\left(\frac{\alpha}{2}\right)}=\frac{2\times\frac{1}{4}}{1-\frac{1}{16}}=\frac{\frac{1}{2}}{\frac{15}{16}}=\frac{16}{30}=\frac{8}{15}$$



$$\frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha}=\frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}} \Rightarrow \boxed{-\frac{16}{105}}$$

۱۰

$$\frac{\frac{\cos\alpha}{\sin\alpha}}{\frac{\sin\alpha}{1}}>0 \qquad \frac{\cos\alpha}{\underbrace{\sin^2\alpha}_{\text{همواره مثبت}}}>0 \xrightarrow{\text{پس}} \cos\alpha>0$$

$$\sin2\alpha=2\sin\alpha\cos\alpha \rightarrow 2\sin\alpha<2\sin\alpha\cos\alpha \xRightarrow{\sin\alpha>0} \cos\alpha>1$$ غ.ق.ق

$$\sin<0 \quad \cos\alpha<1$$ ق.ق (۴)

$\cos +$ / $\sin -$ $\boxed{\text{ناحیه ۴}}$