(۱)

$$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \longrightarrow \frac{1}{|\cos\alpha|} - \tan\alpha = \frac{1-\sin\alpha}{|\cos\alpha|}$$

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یازدهم (خلد)
شماره تلفن: ۲۸

$$\frac{1-1+\sin\alpha}{|\cos\alpha|} = \frac{\sin\alpha}{\cos\alpha} \longrightarrow |\cos\alpha| = \cos\alpha$$

$\cos\alpha\sin\alpha > 0$ ناصفر ↙

$$\cot\alpha = \frac{\cos\alpha}{\sqrt{\frac{1-\cos^2\alpha}{\sin^2\alpha}}} \longrightarrow \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{|\sin\alpha|} \longrightarrow |\sin\alpha| = \sin\alpha$$

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(۲)

$$-\frac{\pi}{12} < \alpha < \frac{5\pi}{12} \longrightarrow -\frac{\pi}{6} < 2\alpha < \frac{5\pi}{6} \longrightarrow -\frac{1}{2} < \sin 2\alpha \leq 1$$

$$-\frac{1}{2} < \frac{m-1}{4} \leq 1$$

$$-2 < m-1 \leq 4 \longrightarrow -1 < m \leq 5$$

مجموعه مقادیر m $\longrightarrow (-1, 5]$

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(۳)

$$\tan\alpha + \cot\alpha = -3 \longrightarrow \frac{\sin^2\alpha + \cos^2\alpha}{\sin\alpha\cos\alpha} = -3$$

$$\sin\alpha\cos\alpha = \frac{-1}{3}$$

$$\sin^3\alpha + \cos^3\alpha = (\sin\alpha + \cos\alpha)(\underbrace{\sin^2\alpha + \cos^2\alpha}_{1} - \underbrace{\sin\alpha\cos\alpha}_{\frac{-1}{3}}) = \frac{-1}{\sqrt{3}} \times \frac{4}{3} = \frac{-4}{3\sqrt{3}}$$

$$(\sin\alpha + \cos\alpha)^2 = \underbrace{\sin^2\alpha + \cos^2\alpha}_{1} + 2\underbrace{\sin\alpha\cos\alpha}_{\frac{-1}{3}}$$

$$\sin\alpha + \cos\alpha = \pm\frac{1}{\sqrt{3}} \quad \leftarrow \quad 1 - \frac{2}{3} = \frac{1}{3}$$

$3\pi < 4\alpha < 4\pi \longrightarrow \frac{3\pi}{4} < \alpha < \pi$

$3\cos\alpha$ ↙

$$-\frac{3\sqrt{3}}{4}$$

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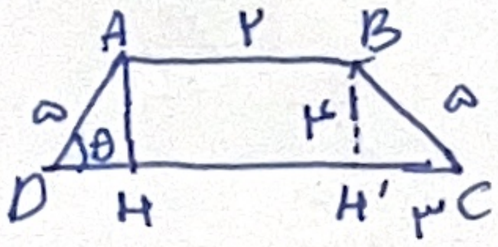
(۴)



$$\cos\theta = \frac{DH}{AD} = \frac{DH}{5} = \frac{4}{10}$$

$$\longrightarrow DH = 3 \qquad AB = HH' = 2$$

$$CH' = 3$$

$$CD = 3 + 3 + 2 = 8 \qquad AD^2 = DH^2 + AH^2 \longrightarrow AH = 4$$

$$S_{\text{ذوزنقه}} = \frac{(AB+CD)\times AH}{2} = \frac{(2+8)\times 4}{2} = 20$$

(۵)

$$\tan(۲۱۵^\circ)\tan(-۱۹۵^\circ) - \sin(۱۰۹۵^\circ)\cos(۲۵۵^\circ) = k\cos^۲ ۱۵$$

$$\tan\left(\frac{۳\pi}{۲}+۱۵\right)\times\tan(-\pi+۱۵) - \sin(۷\pi+۱۵)\times\cos\left(\frac{۳\pi}{۲}-۱۵\right)$$

$$= \cot ۱۵ \times \tan ۱۵ - \sin ۱۵ \times -\sin ۱۵ = -۱ + \sin^۲ ۱۵ = -۱ + (۱-\cos^۲ ۱۵)$$

$$= -\cos^۲ ۱۵ = k\cos^۲ ۱۵ \longrightarrow \boxed{k = -۱}$$

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(۶)

$$A = \sqrt{۳}\cos(۲۱۰)\times\sin(۱۴۴^\circ) - \sqrt{۲}\sin(۱۳۵^\circ)\cos(۱۵۳^\circ)$$

$$\sqrt{۳}\times -\frac{\sqrt{۳}}{۲}\times\sin\left(\frac{۳\pi}{۲}-۲۷\right) - \sqrt{۲}\times\frac{\sqrt{۲}}{۳}\times\cos(\pi-۲۷)$$

$$-\frac{۳}{۲}\times -\cos ۲۷ - ۱ \times -\cos ۲۷ = \frac{۳}{۲}\cos ۲۷ + \cos ۲۷ = \frac{۵}{۲}\cos ۲۷$$

$$\frac{A}{\cos ۲۷} = \frac{\frac{۵}{۲}\cos ۲۷}{\cos ۲۷} = \boxed{\frac{۵}{۲}}$$

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(۷)

$$f\left(\frac{\pi}{۲۴}\right)$$

$$f(x) = ۱۴\cos^۲\left(\frac{۳\pi}{۲x}\right)\cos^۲(۹x)\cos^۲(۱۲x)\cos^۲(۲۴x)$$

$$۱۴\times\cos^۲\left(\frac{\pi}{۱۲}\right)\underbrace{\cos^۲\left(\frac{\pi}{۹}\right)}_{\frac{۱}{۴}}\underbrace{\cos^۲\left(\frac{\pi}{۳}\right)}_{\frac{۳}{۴}}\underbrace{\cos^۲\left(\frac{۲\pi}{۳}\right)}_{\frac{۱}{۴}}$$

$$= ۱۴\times\frac{۲+\sqrt{۳}}{۴}\times\frac{۱}{۴}\times\frac{۳}{۴}\times\frac{۱}{۴} = \frac{۹+۳\sqrt{۳}}{۱۹}$$

$$\cos^۲\left(\frac{۳\pi}{۲۴}\right) = \frac{۱+\cos\frac{\pi}{۹}}{۲} = \frac{۱+\frac{\sqrt{۳}}{۲}}{۲} = \frac{۲+\sqrt{۳}}{۴}$$

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(۳)

$$\sin x\cos x = \frac{۱}{۲}$$

$$(\sin x+\cos x)^۲ = \sin^۲ x + \cos^۲ x + ۲\sin x\cos x$$

$$۱ - \frac{۱}{۲}$$

۳

(۸)

$\frac{1-\sin x}{1+\sin x} = ۴$

$1-\sin x = ۴+۴\sin x$

$\Rightarrow \sin x = -۳$

$\sin x = \frac{-۳}{۵}$

$\sin^۲ x + \cos^۲ x = 1$

$\frac{۹}{۲۵} + \cos^۲ x = 1$

$\begin{cases} \cos x = \frac{-۴}{۵} \checkmark \\ \cos x = \frac{۴}{۵} \text{ غ ق ق} \end{cases}$

$\tan\frac{x}{۲} = \frac{\sin x}{1+\cos x} = \frac{\frac{-۳}{۵}}{1-\frac{۴}{۵}} = \frac{\frac{-۳}{۵}}{\frac{۱}{۵}} = -۳$

---

(۹)

$\frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = \cot\theta \longrightarrow \cot\frac{\theta}{۲} + \cot\frac{\theta}{۲} = ۲\cot\frac{\theta}{۲} = k\cos\frac{\theta}{۲}$

$\frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta} = \tan\frac{\theta}{۲}$

$k = ۲$

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(۱۰)

$\cos\left(\frac{n\pi}{۴}+\alpha\right) = \underbrace{\cos\frac{n\theta}{۴}}_{\frac{-\sqrt{۲}}{۲}}\cos\alpha - \underbrace{\sin\frac{n\pi}{۴}}_{\frac{\sqrt{۲}}{۲}}\sin\alpha$

$\frac{-\sqrt{۲}}{۲} \times \frac{-۷\sqrt{۲}}{۱۵} - \frac{\sqrt{۲}}{۲} \times \frac{\sqrt{۲}}{۱۰} = \frac{۷}{۱۰} - \frac{۱}{۱۰} = \frac{۶}{۱۰} = \frac{۳}{۵}$

$\sin^۲\alpha + \cos^۲\alpha = 1 \longrightarrow \frac{۲}{۱۰۰} + \cos^۲\alpha = 1$

$\begin{cases} \cos\alpha = \frac{+۷\sqrt{۲}}{۱۰} \text{ غ ق ق} \\ \cos\alpha = \frac{-۷\sqrt{۲}}{۱۰} \checkmark \end{cases}$