الهیا جلالی تکلیف شهریور ۲۸

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۱- $\cot\alpha = \frac{\cos\alpha}{|\sin\alpha|} \xrightarrow{\sin\alpha>0}$ ناحیه اول یا دوم

$\frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \longrightarrow \cos\alpha>0$

$\Big\} \to$ ناحیه اول

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۲- $-\frac{\pi}{6} < 2x < \frac{5\pi}{6} \longrightarrow -\frac{1}{2} < \sin 2x \leq 1$

$-\frac{1}{2} < \frac{m-1}{4} \leq 1 \qquad -2 < m-1 \leq 4$

$-1 < m \leq 5$

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۳- $\frac{3\pi}{4} < x < \pi \qquad \tan x + \cot x = \frac{1}{\sin x\cos x} = -3 \qquad \sin x\cos x = -\frac{1}{3}$

$(\sin x+\cos x)^2 = \underbrace{\sin^2 x + \cos^2 x}_{1} + 2\sin x\cos x = 1 - \frac{2}{3} = \frac{1}{3} \Rightarrow \sin x + \cos x = \pm\sqrt{\frac{1}{3}}$

$|\cos x| > |\sin x|, \quad \sin x>0, \ \cos x<0$

$\Rightarrow \sin x + \cos x = -\frac{\sqrt{3}}{3}$

$\sin^3 x + \cos^3 x = (\sin x+\cos x)(\sin^2 x+\cos^2 x - \sin x\cos x) = -\frac{\sqrt{3}}{3} \times \left(1+\frac{1}{3}\right) = -\frac{4\sqrt{3}}{9}$

$\to \frac{1}{\sin^3 x+\cos^3 x} = \frac{-9}{4\sqrt{3}} = \frac{-3\sqrt{3}}{4}$

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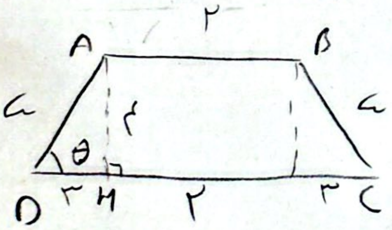

$\sin\theta = 0.8 \qquad AH = 5\times 0.8 = 4$

$DH = 3 \to CD = 8$

$S_{ABCD} = \frac{(AB+CD)AH}{2} = \frac{(2+8)\cancel{4}^{2}}{\cancel{2}} = 20$

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۵-

$-\tan(270+15)\tan(180-15) - \sin(1080+15)\cos(270-15)$

$= +\cot 15 \times(-\tan 15) + \sin 15\sin 15 = -1+\sin^2 15 = -\cos^2 15 \to k=-1$

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۶-

$\sqrt{3}\times -\frac{\sqrt{3}}{2} \times \sin(270-27) - \sqrt{2}\times +\frac{\sqrt{2}}{2}\cos(180-27)$

$+\frac{3}{2}\cos 27 + \cos 27 = \frac{5}{2}\cos 27$ ۵/۲ برابر

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۷- $f\left(\frac{\pi}{24}\right) = 16\cos^2\left(\frac{\pi}{12}\right)\underbrace{\cos^2\left(\frac{\pi}{6}\right)}_{\frac{3}{4}}\underbrace{\cos^2\left(\frac{\pi}{3}\right)}_{\frac{1}{4}}\underbrace{\cos^2\left(\frac{2\pi}{3}\right)}_{\frac{1}{4}} = 16\cos^2\frac{\pi}{12}\times\frac{3}{64}$

$\cos^2\frac{\pi}{12} = \frac{1+\cos\frac{\pi}{6}}{2} = \frac{2+\sqrt{3}}{4} \qquad = \frac{3}{4}\times\frac{2+\sqrt{3}}{4} = \boxed{\frac{6+3\sqrt{3}}{16}}$

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-8

$1-\sin x = 2+2\sin x$ $\quad 3\sin x = -1$ $\quad \sin x = -\frac{3}{5}$ $\quad \cos^2 x = 1-\frac{9}{25} = \frac{16}{25}$ $\quad$ 4

$\cos x = -\frac{4}{5}$

$$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x} = \frac{-\frac{3}{5}}{\frac{1}{5}} = -3$$

-9

$$\frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = \frac{\cancel{2}\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cancel{2}\sin^2\frac{\theta}{2}} + \frac{\cancel{2}\cos^2\frac{\theta}{2}}{\cancel{2}\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = 2\cot\frac{\theta}{2}$$

$k=2$ $\quad$ 2

-10

$\cos^2\alpha = 1-\frac{2}{100} = \frac{98}{100}$ $\quad \cos\alpha = -\frac{7\sqrt{2}}{10}$

$$\cos\left(\frac{11\pi}{4}+\alpha\right) = \underbrace{\cos\frac{11\pi}{4}}_{-\frac{\sqrt{2}}{2}}\cos\alpha - \underbrace{\sin\frac{11\pi}{4}}_{\frac{\sqrt{2}}{2}}\sin\alpha = -\frac{\sqrt{2}}{2} \times -\frac{7\sqrt{2}}{10} - \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{10}$$

1

$$\frac{7}{10} - \frac{1}{10} = \frac{6}{10}$$