المپیادهای تکلیف شهریور ۲۸

۱-

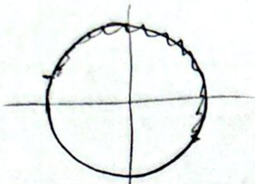
$\cot\alpha = \dfrac{\cos\alpha}{|\sin\alpha|} \xrightarrow{\sin\alpha > 0}$ ناحیه اول یا دوم

$\dfrac{1}{|\cos\alpha|} - \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{1-\sin\alpha}{|\cos\alpha|} \longrightarrow \cos\alpha > 0$

$\rightarrow$ ناحیه اول

۲-

$-\dfrac{\pi}{6} < 2x < \dfrac{5\pi}{6}$



$\longrightarrow -\dfrac{1}{2} < \sin 2x \le 1$

$-\dfrac{1}{2} < \dfrac{m-1}{4} \le 1 \qquad -2 < m-1 \le 4$

$-1 < m \le 5$

۳-

$\dfrac{3\pi}{4} < x < \pi \qquad \tan x + \cot x = \dfrac{1}{\sin x\cos x} = -3 \qquad \sin x\cos x = -\dfrac{1}{3}$

$(\sin x + \cos x)^2 = \underbrace{\sin^2 x + \cos^2 x}_{1} + 2\sin x\cos x = 1 - \dfrac{2}{3} = \dfrac{1}{3} \Rightarrow \sin x + \cos x = \pm\sqrt{\dfrac{1}{3}}$

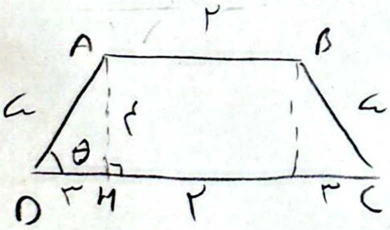
$|\cos x| > |\sin x|, \quad \sin x > 0, \ \cos x < 0$

$\Rightarrow \sin x + \cos x = -\dfrac{\sqrt{3}}{3}$

$\sin^3 x + \cos^3 x = (\sin x + \cos x)(\sin^2 x + \cos^2 x - \sin x\cos x) = -\dfrac{\sqrt{3}}{3} \times \left(1 + \dfrac{1}{3}\right) = -\dfrac{4\sqrt{3}}{9}$

$\rightarrow \dfrac{1}{\sin^3 x + \cos^3 x} = \dfrac{-9}{4\sqrt{3}} = \dfrac{-3\sqrt{3}}{4}$

۴-



$\sin\theta = 0.8 \qquad AH = 5 \times 0.8 = 4$

$DH = 3 \rightarrow CD = 8$

$S_{ABCD} = \dfrac{(AB+CD)AH}{2} = \dfrac{(2+8)\cancel{4}^{2}}{\cancel{2}} = 20$

۵-

$-\tan(270+15)\tan(180-15) - \sin(1080+15)\cos(270-15)$

$= +\cot 15 \times (-\tan 15) + \sin 15 \sin 15 = -1 + \sin^2 15 = -\cos^2 15 \rightarrow k = -1$

۶-

$\sqrt{3} \times -\dfrac{\sqrt{3}}{2} \times \sin(270-27) - \cancel{\sqrt{2}} \times +\dfrac{\sqrt{2}}{2}\cos(180-27)$

$+\dfrac{3}{2}\cos 27 + \cos 27 = \dfrac{5}{2}\cos 27$

$\dfrac{5}{2}$ برابر

۷-

$f\left(\dfrac{\pi}{24}\right) = 16\cos^2\left(\dfrac{\pi}{12}\right)\underbrace{\cos^2\left(\dfrac{\pi}{6}\right)}_{\frac{3}{4}}\underbrace{\cos^2\left(\dfrac{\pi}{3}\right)}_{\frac{1}{4}}\underbrace{\cos^2\left(\dfrac{2\pi}{3}\right)}_{\frac{1}{4}} = \cancel{16}\cos^2\dfrac{\pi}{12} \times \dfrac{3}{\cancel{64}_{4}}$

$\cos^2\dfrac{\pi}{12} = \dfrac{1+\cos\frac{\pi}{6}}{2} = \dfrac{2+\sqrt{3}}{4}$

$= \dfrac{3}{4} \times \dfrac{2+\sqrt{3}}{4} = \boxed{\dfrac{6+3\sqrt{3}}{16}}$

-8

$1 - \sin x = 4 + 4\sin x \qquad 5\sin x = -3 \qquad \sin x = -\frac{3}{5} \quad , \quad \cos^2 x = 1 - \frac{9}{25} = \frac{16}{25}$

$\cos x = -\frac{4}{5}$

$$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x} = \frac{-\frac{3}{\cancel{5}}}{\frac{1}{\cancel{5}}} = -3$$

-9

$$\frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = \frac{\cancel{2}\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cancel{2}\cancel{\sin^2\frac{\theta}{2}}} + \frac{\cancel{2}\cos^2\frac{\theta}{2}}{\cancel{2}\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = 2\cot\frac{\theta}{2}$$

$k = 2$

-10

$\cos^2\alpha = 1 - \frac{2}{100} = \frac{98}{100} \qquad \cos\alpha = -\frac{7\sqrt{2}}{10}$

$$\cos\left(\frac{11\pi}{4}+\alpha\right) = \underbrace{\cos\frac{11\pi}{4}}_{-\frac{\sqrt{2}}{2}}\cos\alpha - \underbrace{\sin\frac{11\pi}{4}}_{\frac{\sqrt{2}}{2}}\sin\alpha = -\frac{\sqrt{2}}{2} \times -\frac{7\sqrt{2}}{10} - \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{10}$$

$$\frac{7}{10} - \frac{1}{10} = \frac{6}{10}$$