به نام خداوند دانش

دینا صابری [illegible]

۱۹,۲۵

①

$$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|}$$

$$\cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} = \frac{\cos\alpha}{|\sin\alpha|}$$

$$\Rightarrow \sin\alpha > 0$$

$$\Rightarrow \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \Rightarrow \cos\alpha > 0$$

ناحیه اول مثلثاتی

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②

$$\sin 2x = \frac{m-1}{4}$$

$$\frac{-\pi}{12} < x < \frac{5\pi}{12} \Rightarrow -\frac{\pi}{6} < 2x < \frac{5\pi}{6} \qquad -\frac{1}{2} < \sin 2x \le 1$$

$$\Rightarrow -\frac{1}{2} < \frac{m-1}{4} \le 1$$

$$\Rightarrow -\frac{1}{2} < \frac{m-1}{4} \Rightarrow -2 < m-1 \Rightarrow \boxed{-1 < m}$$

$$\Rightarrow \frac{m-1}{4} \le 1 \Rightarrow m-1 \le 4 \Rightarrow \boxed{m \le 5}$$

$$\Rightarrow -1 < m \le 5$$

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③

$$\tan x + \cot x = -3$$

$$3\pi < 4x < 4\pi \Rightarrow \frac{3\pi}{4} < x < \pi \Rightarrow -\sin x\cos x > 0$$

$$\sin^3 x + \cos^3 x = S^3 - 3PS$$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x\cos x} = -3 \Rightarrow \sin^2 x + \cos^2 x = -3\sin x\cos x$$

$$S^2 - 2P$$

$$(\sin x + \cos x)^2 - 2\sin x\cos x = -3\sin x\cos x$$

$$\Rightarrow (\sin x + \cos x)^2 = -\sin x\cos x$$

$$1 = -3\sin x\cos x \Rightarrow \underbrace{\sin x\cos x}_{P} = -\frac{1}{3}$$

$$\boxed{S^2 = -P} \Rightarrow S^3 = -P\sqrt{-P}$$

$$S = \sqrt{-P}$$

$$= -P\sqrt{-P} - 3P\sqrt{-P} = -4P\sqrt{-P} = -4 \times -\frac{1}{3}\sqrt{\frac{1}{3}} = \boxed{\frac{4}{3}\sqrt{\frac{1}{3}}}$$

$$\Rightarrow \frac{1}{\sin^3 x + \cos^3 x} = \frac{1}{\frac{4}{3}\sqrt{\frac{1}{3}}} = \boxed{\frac{3}{4}\sqrt{3}}$$

۱,۷۵

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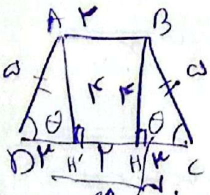
④



$$\cos\theta = \frac{6}{10} \Rightarrow \sin\theta = \frac{8}{10}$$

$$CH = \frac{6}{10} \times 5 = 3$$

۸ = قاعده بزرگ

ارتفاع ذوزنقه $= BH = 0.8 \times 5 = 4$

مساحت ذوزنقه $= \frac{1}{2}$ (ارتفاع) (قاعده بزرگ + قاعده کوچک)

$$= \frac{1}{2} \times 4(\underbrace{2+8}_{10}) = 20$$

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⑤

$$\tan(285^\circ)\tan(-165) - \sin(1095)\cos(255) = k\cos^2 15$$

$$\tan(\frac{3\pi}{2}+15)\tan(15-\pi) - \sin(6\pi+15)\cos(\frac{3\pi}{2}-15)$$

$$-\cot(15)(\tan 15) + (\sin(15))(\sin(15))$$

$$-1 \qquad +\sin^2 15$$

$$-1 + \sin^2 15 = -1 + \cos^2 15$$

$$1 - \cos^2 15$$

$$= -1 + 1 - \cos^2 15 = -\cos^2 15$$

$$\boxed{k = -1}$$

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۶)

$$\sqrt{۳}\,\underbrace{\cos(۲۱۰)}_{-\frac{\sqrt{۳}}{۲}}\ \sin\left(\underbrace{\tfrac{۳\pi}{۲}-۲۷}_{۲۴۳}\right)-\sqrt{۲}\,\underbrace{\sin(۱۳۵)}_{\frac{\sqrt{۲}}{۲}}\ \underbrace{\cos(۱۵۳)}_{\cos(\pi-۲۷)}$$

$$-\frac{۳}{۲}(-\cos ۲۷)-\sqrt{۲}\left(\frac{\sqrt{۲}}{۲}\right)(-\cos ۲۷)$$

$$=\frac{۳}{۲}\cos(۲۷)+\cos ۲۷=\frac{۵}{۲}\cos ۲۷ \Rightarrow \frac{۵}{۲}\ \text{برابر}$$

(۱)

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۷)

$$f\left(\frac{\pi}{۲۴}\right)=۱۶\cos^۲\left(\frac{\pi}{۱۲}\right)\cos^۲\left(\frac{\pi}{۶}\right)\cos^۲\left(\frac{\pi}{۴}\right)\cos^۲\left(\frac{۳\pi}{۴}\right)$$

$$۴(۲+\sqrt{۳})\times\frac{۳}{۴}\times\frac{۱}{۴}\times\frac{۱}{۴}=\boxed{\frac{۶+۳\sqrt{۳}}{۱۶}}$$

$$\cos^۲\frac{\pi}{۱۲}=\frac{۱+\cos\frac{\pi}{۶}}{۲}=\frac{۱+\frac{\sqrt{۳}}{۲}}{۲}=\frac{۲+\sqrt{۳}}{۴}$$

(۱/۵)

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۸)

$$\frac{۱-\cos x}{\sin x}=\tan\frac{x}{۲}=\frac{\frac{۹}{۵}}{-\frac{۳}{۵}}=\boxed{-۳}\qquad \frac{۱-\sin x}{۱+\sin x}=۴ \Rightarrow$$

ربع سوم

$$۴+۴\sin x=۱-\sin x$$

$$۵\sin x=-۳ \Rightarrow \boxed{\sin x=-\frac{۳}{۵}}$$

$$\Rightarrow \underbrace{\sin^۲ x}_{\frac{۹}{۲۵}}+\cos^۲ x=۱ \Rightarrow \cos^۲ x=\frac{۱۶}{۲۵}$$

ربع سوم $\rightarrow$ $\boxed{-\frac{۴}{۵}}=\cos x$

(۱)

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۹)

$$\underbrace{\frac{\sin\theta}{۱-\cos\theta}}_{\cot\frac{\theta}{۲}}+\underbrace{\frac{۱+\cos\theta}{\sin\theta}}_{\cot\frac{\theta}{۲}}=۲\cot\frac{\theta}{۲}=k\cot\frac{\theta}{۲}\Rightarrow k=۲$$

(۱)

CamScanner

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(۱۱) ربع دوم

۹

$\sin\alpha = \frac{\sqrt{2}}{10}$

$\cos\left(\frac{11\pi}{4}+\alpha\right) = \cos\frac{11\pi}{4}\cdot\cos\alpha - \sin\frac{11\pi}{4}\cdot\sin\alpha$

$\frac{11\pi}{4} = \frac{3\pi}{4}$

$= -\frac{\sqrt{2}}{2}\cos\alpha - \frac{\sqrt{2}}{2}\cdot\sin\alpha = -\frac{\sqrt{2}}{2}(\cos\alpha+\sin\alpha)$

$\cos^2\alpha+\underbrace{\sin^2\alpha}_{\frac{2}{100}} = 1 \Rightarrow \cos^2\alpha = \frac{98}{100} \Rightarrow \cos\alpha = -\frac{\sqrt{98}}{10}$ (نامعتبر) $\to -7\sqrt{2}$

$-\frac{\sqrt{2}}{2}\underbrace{\left(\frac{\sqrt{2}}{10} - \frac{7\sqrt{2}}{10}\right)}_{-\frac{6\sqrt{2}}{10}} = \frac{3}{5}$