به نام خداوند دانش

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نام مبحث: مثلثات

①

$$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|}$$

$$\Rightarrow \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \Rightarrow \cos\alpha > 0$$

$$\cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} = \frac{\cos\alpha}{|\sin\alpha|}$$

$$\Rightarrow \sin\alpha > 0$$

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②

$$\sin 2x = \frac{m-1}{4}$$

$$\frac{-\pi}{12} < x < \frac{5\pi}{12} \Rightarrow -\frac{\pi}{6} < 2x < \frac{5\pi}{6} \qquad -\frac{1}{2} < \sin 2x \le 1$$

$$\Rightarrow -\frac{1}{2} < \frac{m-1}{4} \le 1$$

$$\Rightarrow -\frac{1}{2} < \frac{m-1}{4} \Rightarrow -2 < m-1 \Rightarrow \boxed{-1 < m}$$

$$\Rightarrow \frac{m-1}{4} \le 1 \Rightarrow m-1 \le 4 \Rightarrow \boxed{m \le 5}$$

$$\cap \Rightarrow -1 < m \le 5$$

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③

$$\tan x + \cot x = -3$$

$$3\pi < 4x < 4\pi \Rightarrow \frac{3\pi}{4} < x < \pi \Rightarrow -\sin x\cos x > 0$$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = -3 \Rightarrow \underbrace{\sin^2 x + \cos^2 x}_{1} = -3\sin x\cos x$$

$$\sin^3 x + \cos^3 x = S^3 - 3PS$$

$$= -P\sqrt{-P} - 3P\sqrt{-P} = -4P\sqrt{-P} = -4 \times -\frac{1}{3}\sqrt{\frac{1}{3}} = \boxed{\frac{4}{3}\sqrt{\frac{1}{3}}}$$

$$\Rightarrow \frac{1}{\sin^3 x + \cos^3 x} = \frac{1}{\frac{4}{3}\sqrt{\frac{1}{3}}} = \boxed{\frac{3}{4}\sqrt{3}}$$

$$S^2 - 2P$$

$$(\sin x + \cos x)^2 - 2\sin x\cos x = -3\sin x\cos x$$

$$\Rightarrow (\sin x + \cos x)^2 = -\sin x\cos x$$

$$1 = -3\sin x\cos x \Rightarrow \underbrace{\sin x\cos x}_{P} = -\frac{1}{3}$$

$$\boxed{S^2 = -P} \Rightarrow S^3 = -P\sqrt{-P}$$

$$S = \sqrt{-P}$$

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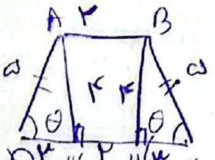
④



$$\cos\theta = \frac{6}{10} \Rightarrow \sin\theta = \frac{8}{10}$$

$$CH = \frac{6}{10} \times 5 = 3$$

ارتفاع ذوزنقه $= BH = 0/8 \times 5 = 4$ (مثلث طلایی)

مساحت ذوزنقه $= \frac{1}{2}$ (ارتفاع) (قاعده بزرگ + قاعده کوچک)

$$= \frac{1}{2} \times 4 (\underbrace{2+8}_{10}) = 20$$

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⑤

$$\tan(285^\circ)\tan(-165) - \sin(105)\cos(255) = k\cos^2 15$$

$$\tan\left(\frac{3\pi}{2}+15\right)\tan(15-\pi) - \sin\left(\frac{\pi}{2}+15\right)\cos\left(\frac{3\pi}{2}-15\right)$$

$$-\cot(15)(\tan 15) + (\sin(15) \times \sin(15))$$

$$-1 \qquad +\sin^2 15$$

$$-1+\sin^2 15 = -1 + \cos^2 15$$

$$1-\cos^2 15$$

$$= -1+1-\cos^2 15 = -\cos^2 15$$

$k = ?$ $\boxed{k=-1}$

بنام خداوند بخشنده

(۱۱) α ربع دوم

$\sin\alpha = \frac{\sqrt{2}}{10}$

$$\cos\left(\frac{11\pi}{4}+\alpha\right) = \cos\frac{11\pi}{4}\cdot\cos\alpha - \sin\frac{11\pi}{4}\cdot\sin\alpha \qquad \frac{11\pi}{4} = \frac{3\pi}{4}$$

$$= -\frac{\sqrt{2}}{2}\cos\alpha - \frac{\sqrt{2}}{2}\cdot\sin\alpha = -\frac{\sqrt{2}}{2}(\cos\alpha + \sin\alpha)$$

$$\cos^2\alpha + \underbrace{\sin^2\alpha}_{\frac{2}{100}} = 1 \Rightarrow \cos^2\alpha = \frac{98}{100} \Rightarrow \cos\alpha = -\frac{\sqrt{98}}{10}$$

ناصحیح

$\sqrt{98} \rightarrow 7\sqrt{2}$, i.e. $\cos\alpha = \frac{-7\sqrt{2}}{10}$

$$-\frac{\sqrt{2}}{2}\underbrace{\left(\frac{\sqrt{2}}{10} - \frac{7\sqrt{2}}{10}\right)}_{\frac{-6\sqrt{2}}{10}} = \frac{3}{5}$$