نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

۱

$$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \rightarrow \frac{1}{|\cos\alpha|} - \tan\alpha = \frac{1-\sin\alpha}{|\cos\alpha|}$$

$$\frac{1-(1+\sin\alpha)}{|\cos\alpha|} = \frac{\sin\alpha}{\cos\alpha} \rightarrow |\cos\alpha| = \cos\alpha \qquad \cos\alpha \geq 0$$

$$\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} \rightarrow \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{|\sin\alpha|} \rightarrow |\sin\alpha| = \sin\alpha$$

۲

$$-\frac{\pi}{12} < x < \frac{5\pi}{12} \rightarrow -\frac{\pi}{6} < 2x < \frac{5\pi}{6} \rightarrow -\frac{1}{2} < \sin 2x \leq 1$$

$$-\frac{1}{2} < \frac{m-1}{4} \leq 1 \rightarrow -2 < m-1 \leq 4 \rightarrow -1 < m \leq 5$$

مجموعه مقادیر $\rightarrow (-1, 5]$

۳

$$\tan x + \cot x = -3 \rightarrow \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = -2 \rightarrow \sin x \cos x = -\frac{1}{3}$$

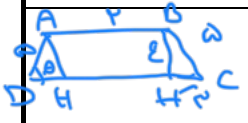
$$\sin^2 x + \cos^2 x = (\sin x + \cos x)(\sin^2 x + \cos^2 x - \sin x \cos x)$$

$$= -\frac{1}{\sqrt{3}} \times \frac{4}{3} = \frac{-4}{3\sqrt{3}} \rightarrow (\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2\sin x \cos x$$

$$\rightarrow 1 - \frac{2}{3} = \frac{1}{3} \rightarrow \sin x + \cos x = \pm\frac{1}{\sqrt{3}} \rightarrow \pi < 4x < 4\pi \rightarrow \frac{3\pi}{4} < 2x < \frac{5\pi}{4}$$

$$\rightarrow -1 < \cos x < -\frac{\sqrt{2}}{2} \quad , \quad 0 < \sin x < \frac{\sqrt{2}}{2} \rightarrow \sin x + \cos x < 0 \rightarrow \frac{1}{\sin x + \cos x} = -\frac{\sqrt{3}}{3}$$

۴



$$\cos\theta = \frac{DH}{AD} = \frac{DH}{5} = \frac{4}{10} \rightarrow DH = 3 \rightarrow CH' = 3$$

$$AB = HH' = 2$$

$$\rightarrow CD = 3+2+3 = 8 \quad , \quad AD^2 = DH^2 + AH^2 \rightarrow AH = 4$$

$$S_{\text{ذوزنقه}} = \frac{(AB+CD)\times AH}{2} = \frac{(2+8)\times 4}{2} = 20$$

۵

$$\tan(2\pi\alpha)\tan(-14\alpha) - \sin(10\alpha)\cos(2\alpha) =$$

$$\tan\left(\frac{2\pi}{2} + 12\alpha\right) \times \tan(-2\pi + 12\alpha) - \sin(4\pi + 12\alpha) \times \cos\left(3\pi - \frac{\pi}{2} - 12\alpha\right)$$

$$= \cot 12\alpha \times \tan 12\alpha - \sin 12\alpha \times \sin 12\alpha = 1 + \sin^2 12\alpha = 1 + (1 - \cos^2 12\alpha)$$

$$= -\cos^2 12\alpha = k \cos^2 12\alpha \rightarrow k = -1$$

۶	$A = \sqrt{3}\cos(210)\times\sin(240) - \sqrt{2}\sin(135)\cos(135)$ $= \sqrt{2}\times -\frac{\sqrt{3}}{2}\times\sin(\frac{3\pi}{2}-\pi) - \sqrt{2}\times\frac{1}{\sqrt{2}}\times\cos(\pi-2\pi)$ $= -\frac{\sqrt{6}}{2}\times -\cos\pi - 1\times -\cos 2\pi = \frac{\sqrt{6}}{2}\cos\pi + \cos 2\pi = -\frac{\sqrt{6}}{2}\cos 2\pi$ $\frac{A}{\cos 2\pi} = \frac{-\frac{\sqrt{6}}{2}\cos 2\pi}{\cos 2\pi} =$ (۳/۵)
۷	$f(\frac{5\pi}{12}) = 14\cos^2(\frac{\pi}{12})\times\underbrace{\cos^2(\frac{\pi}{12})}_{\frac{1}{2}}\times\underbrace{\cos^2(\frac{\pi}{12})}_{\frac{3}{4}}\times\underbrace{\cos^2(\frac{5\pi}{12})}_{\frac{1}{4}}$ $= 14\times\frac{2+\sqrt{3}}{4}\times\frac{1}{2}\times\frac{3}{4}\times\frac{1}{4} = \frac{21(2+\sqrt{3})}{16}$ $\cos^2(\frac{\pi}{12}) = \frac{1+\cos\frac{\pi}{6}}{2} = \frac{1+\frac{\sqrt{3}}{2}}{2} = \frac{\frac{2+\sqrt{3}}{2}}{2} = \frac{2+\sqrt{3}}{4}$
۸	$\frac{1-\sin\alpha}{1+\sin\alpha} = 3 \to 1-\sin\alpha = 3+3\sin\alpha \to 2\sin\alpha = -2 \to \sin\alpha = -\frac{1}{2}$ $\sin^2\alpha+\cos^2\alpha = 1 \to \frac{1}{4}+\cos^2\alpha = 1 \begin{cases}\cos\alpha = \frac{\sqrt{3}}{2} \\ \cos\alpha = -\frac{\sqrt{3}}{2}\end{cases}$ قبول $\tan\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha} = \frac{-\frac{1}{2}}{1+\frac{\sqrt{3}}{2}} = \frac{-1}{2+\sqrt{3}} =$ (۳)
۹	$\frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$ $\frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta} = \tan\frac{\theta}{2}$ $\cot\frac{\theta}{2} + \cot\frac{\theta}{2}$ $= 2\cot\frac{\theta}{2} = k\cot\frac{\theta}{2} \to k = 2$
۱۰	$\cos(\frac{3\pi}{4}+\alpha) = \underbrace{\cos\frac{3\pi}{4}}_{-\frac{\sqrt{2}}{2}}\cos\alpha - \underbrace{\sin\frac{3\pi}{4}}_{\frac{\sqrt{2}}{2}}\sin\alpha$ $= -\frac{\sqrt{2}}{2}\times -\frac{\sqrt{10}}{5} - \frac{\sqrt{2}}{2}\times\frac{\sqrt{15}}{5} = \frac{\sqrt{5}}{5} - \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5} - \frac{\sqrt{5}}{5} =$ (۰) $\sin^2\alpha+\cos^2\alpha = 1 \to \frac{2}{5}+\cos^2\alpha = 1 \begin{cases}\cos\alpha = +\frac{\sqrt{10}}{5} \\ \cos\alpha = -\frac{\sqrt{10}}{5}\end{cases}$ قبول ✓