لینا ابراهیمی شماره ۲۸ یازدهم دفتر C

۱-

$$\frac{1}{\sqrt{\cos^2\alpha}}-\frac{1}{\cot\alpha}=\frac{1-\sin\alpha}{|\cos|}\Rightarrow\frac{1}{|\cos\alpha|}-\tan\alpha=\frac{1-\sin\alpha}{|\cos\alpha|}$$

$$\frac{1-1+\sin\alpha}{|\cos\alpha|}=\frac{\sin\alpha}{\cos\alpha}\Rightarrow|\cos\alpha|=\cos\alpha$$

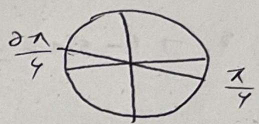
$\cos\alpha$ $\sin\alpha$ مثبت هستند

$\boxed{\text{ناحیه ۱}}$

$$\cot\alpha=\frac{\cos\alpha}{\sqrt{\underbrace{1-\cos^2}_{\sin^2\alpha}}}\Rightarrow\frac{\cos\alpha}{\sin\alpha}=\frac{\cos\alpha}{|\sin\alpha|}\Rightarrow|\sin\alpha|=\sin\alpha$$

۲-

$$-\frac{\pi}{12}<x<\frac{5\pi}{12}\rightarrow-\frac{\pi}{6}<2x<\frac{5\pi}{6}\rightarrow-\frac{1}{2}<\sin 2x\le 1$$



$$-\frac{1}{2}<\frac{m-1}{4}\le 1\rightarrow-2<m-1\le 4\rightarrow-1<m\le 5$$

مجموعه مقادیر $m \Rightarrow (-1,5]$

۳-

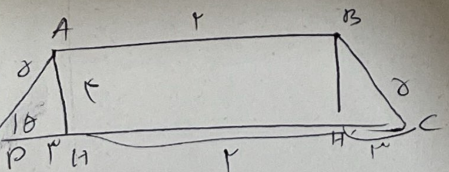
$$\tan x+\cot x=-3\rightarrow\frac{\overbrace{\sin^2x+\cos^2x}^{1}}{\sin x\cos x}=-3\Rightarrow\sin x\cos x=-\frac{1}{3}$$

$$\sin^3x+\cos^3x=(\sin x+\cos x)(\underbrace{\sin^2x+\cos^2x}_{1}\underbrace{-\sin x\cos x}_{-\frac{1}{3}})=-\frac{1}{\sqrt{3}}\times\frac{4}{3}=\frac{-4}{3\sqrt{3}}$$

$$(\sin x+\cos x)^2=\sin^2x+\cos^2x+2\sin x\cos x=1-\frac{2}{3}=\frac{1}{3}\rightarrow\sin x+\cos x=\overset{\text{غ ق ق}}{\pm}\frac{1}{\sqrt{3}}$$

$$\rightarrow\sin x+\cos x<0\qquad\frac{1}{\sin^3x+\cos^3x}=\frac{1}{-\frac{4}{3\sqrt{3}}}=\boxed{\frac{-3\sqrt{3}}{4}}$$

۴-



$$\cos\theta=\frac{DH}{AD}=\frac{DH}{5}=\frac{6}{10}\rightarrow DH=3$$

$$CH'=3\qquad AB=HH'=2$$

$$CD=3+3+2=8\qquad AD^2=DH^2+AH^2\rightarrow 5^2=3^2+AH^2\rightarrow AH=4$$

$$S_{\text{ذوزنقه}}=\frac{(AB+CD)\times AH}{2}=\frac{(2+8)\times 4}{2}=20$$

۵-

$$\tan(285)\tan(-165)-\sin(1095)\cos(255)=\tan(\tfrac{3\pi}{2}+15)\times\tan(-\pi+15)-\sin(6\pi+15)\times\cos(\tfrac{3\pi}{2}-15)$$

$$=-\cot 15\times\tan 15-\sin 15\times-\sin 15=-1+\sin^2 15=-1+(1-\cos^2 15)=-\cos^2 15=k\cos^2 15$$

$$\Rightarrow\boxed{k=-1}$$

۶-

$A=\sqrt{3}\cos(210)\times\sin(243)-\sqrt{2}\sin(135)\cos(153)$

$=\sqrt{3}\times-\frac{\sqrt{3}}{2}\times\sin\left(\frac{3\pi}{2}-27\right)-\sqrt{2}\times\frac{\sqrt{2}}{2}\times\cos(\pi-27)=-\frac{3}{2}\times-\cos 27-1\times-\cos 27=$

$\frac{3}{2}\cos 27+\cos 27=\frac{5}{2}\cos 27$

$\frac{A}{\cos 27^\circ}=\frac{\frac{5}{2}\cos 27}{\cos 27}=\frac{5}{2}$

۷-

$f\left(\frac{\pi}{24}\right)=16\cos^2\left(\frac{\pi}{12}\right)\times\underbrace{\cos^2\left(\frac{\pi}{6}\right)}_{\frac{1}{4}}\times\underbrace{\cos^2\left(\frac{\pi}{3}\right)}_{\frac{3}{4}}\times\underbrace{\cos^2\left(\frac{2\pi}{3}\right)}_{\frac{1}{4}}=16\times\frac{2+\sqrt{3}}{4}\times\frac{1}{4}\times\frac{3}{4}\times\frac{1}{4}=\boxed{\frac{6+3\sqrt{3}}{16}}$

$\cos^2\left(\frac{\pi}{12}\right)=\frac{1+\cos\frac{\pi}{6}}{2}=\frac{1+\frac{\sqrt{3}}{2}}{2}=\frac{\frac{2+\sqrt{3}}{2}}{2}=\frac{2+\sqrt{3}}{4}$

۸-

$\frac{1-\sin x}{1+\sin x}=4\rightarrow 1-\sin x=4+4\sin x\rightarrow 5\sin x=-3\rightarrow\sin x=-\frac{3}{5}$

$\sin^2+\cos^2=1\rightarrow\frac{9}{25}+\cos^2 x=1\rightarrow\cos x=\overset{\text{ربع ۴}}{\underset{\text{ربع ۳}}{\pm}}\frac{4}{5}$

$\tan\frac{x}{2}=\frac{\sin x}{1+\cos x}=\frac{-\frac{3}{5}}{1-\frac{4}{5}}=\frac{-\frac{3}{5}}{\frac{1}{5}}=-3$

۹-

$\frac{\sin\theta}{1-\cos\theta}+\frac{1+\cos\theta}{\sin\theta}\longrightarrow\frac{\sin\theta}{1+\cos\theta}=\frac{1-\cos\theta}{\sin\theta}=\tan\frac{\theta}{2}$

$\cot\frac{\theta}{2}+\cot\frac{\theta}{2}=2\cot\frac{\theta}{2}=k\cot\frac{\theta}{2}\rightarrow\boxed{k=2}$

۱۰-

$\cos\left(\frac{11\pi}{4}+\alpha\right)=\overbrace{\cos\frac{11\pi}{4}}^{-\frac{\sqrt{2}}{2}}\cos\alpha-\overbrace{\sin\frac{11\pi}{4}}^{\frac{\sqrt{2}}{2}}\sin\alpha$

$=-\frac{\sqrt{2}}{2}\times\frac{-7\sqrt{2}}{10}-\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{10}=\frac{7}{10}-\frac{1}{10}=\frac{6}{10}=\frac{3}{5}$

$\sin^2\alpha+\cos^2\alpha=1\rightarrow\frac{2}{100}+\cos^2\alpha=1\rightarrow\cos\alpha=\overset{\text{ربع ۴}}{\underset{\text{ربع ۳}}{\pm}}\frac{7\sqrt{2}}{10}$