نام و نام خانوادگی: آیسین عقیلی   تکلیف: ۲۸   کلاس: نخبه یازدهم ریاضی

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$\Rightarrow \frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|}$ ($\frac{1}{\cot\alpha}$: $\tan\alpha$) $\to \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \to \frac{\sin\alpha}{\cos\alpha} = \frac{1}{|\cos\alpha|} - \frac{1-\sin\alpha}{|\cos\alpha|}$

$\frac{\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{|\cos\alpha|} \to \cos\alpha > 0$

$\to \cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} = \frac{\cos\alpha}{|\sin\alpha|} \to \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{|\sin\alpha|} \to \sin\alpha > 0$ ناحیه α ①

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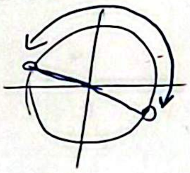
$-\frac{\pi}{12} < x < \frac{5\pi}{12} \to -\frac{\pi}{6} < 2x < \frac{5\pi}{6}$



$-\frac{1}{2} < \sin 2x \leq 1$

$\sin 2x = \frac{m-1}{4}$

$-\frac{1}{2} < \frac{m-1}{4} \leq 1 \quad)\times 4$

$-2 < m-1 \leq 4$

$-1 < m \leq 5$

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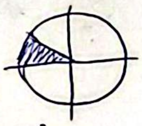
$\tan x + \cot x = -3 \to \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{1}{\sin x \cos x} = -3 \to \sin x\cos x = -\frac{1}{3}$

$3\pi < 4x < 4\pi$

$\to \frac{1}{\sin^3 x + \cos^3 x} = \frac{1}{(\sin x+\cos x)(1-\sin x\cos x)} = \frac{3}{4} \times \frac{1}{\sin x + \cos x}$



از طرفی $(\sin x + \cos x)^2 = 1 + 2(\sin x\cos x) = 1 - \frac{2}{3} = \frac{1}{3}$

$\sin x + \cos x = \frac{\sqrt{3}}{3}$ ✗ چون $\frac{3\pi}{4} < x < \pi$

$\sin x + \cos x = -\frac{\sqrt{3}}{3}$ ✓ چون در این ناحیه $|\cos x| > |\sin x|$ است پس $\sin x + \cos x$ منفی می‌باشد

جواب نهایی: $\frac{3}{4} \times \frac{1}{\sin x + \cos x} = \frac{3}{4} \times \frac{-3}{\sqrt{3}} = -\frac{3\sqrt{3}}{4}$

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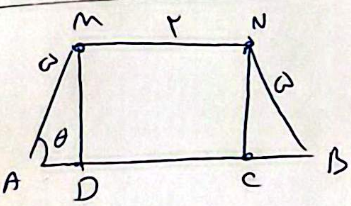
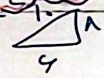


$\cos\theta = 0.6 \to$



$\sin\theta = 0.8$

$MD =$ ارتفاع $= 5\cdot\sin\theta = 5\times 0.8 = 4$

طول $AB = CD + 2\times AD = 2 + 2(5\times\cos\theta) = 8$

$S_{\text{ذوزنقه}} = \frac{\text{ارتفاع} \times \text{مجموع ۲ قاعده}}{2} = \frac{(2+8)\times 4}{2} = 20$

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$\tan(285°)\tan(-165°) - \sin(105°)\cos(255°) = k\cos^2 15$

$\tan(\frac{3\pi}{2}+15)\tan(-\pi+15) - \sin(15)\cos(\frac{3\pi}{2}-15)$

$-\cot(15)\tan(15) - \sin(15)\cdot(-\sin(15)) = -\cot(15)\tan(15) + \sin^2(15) = \sin^2(15) - 1$

$k\cos^2 15 = -\cos^2(15)$

$k = -1$

$A = \sqrt{3}\underbrace{\cos(210°)}_{-\frac{\sqrt{3}}{2}}\sin(243°) - \sqrt{2}\underbrace{\sin(135°)}_{\frac{\sqrt{2}}{2}}\cos(153°) = -\frac{3}{2}\sin(243°) - \cos(153°)$ ۶

$= -\frac{3}{2}\sin\left(\frac{3\pi}{2} - 27°\right) - \cos(\pi - 27) = +\frac{3}{2}\cos(27) + \cos(27) = \frac{\frac{5}{2}\cos(27)}{\cos(27)}$; $\frac{5}{2}$ برابر

$f(x) = 16\cos^2(3x)\cos^2(6x)\cos^2(12x)\cos^2(24x)$

$f(\frac{\pi}{36}) = 16\cos^2(\frac{\pi}{12})\underbrace{\cos^2(\frac{\pi}{6})}_{(\frac{\sqrt{3}}{2})^2}\underbrace{\cos^2(\frac{\pi}{3})}_{(\frac{1}{2})^2}\underbrace{\cos^2(\frac{2\pi}{3})}_{(-\frac{1}{2})^2} = 16 \times \frac{3}{4} \times \frac{1}{4} \times \frac{1}{4}\cos^2(\frac{\pi}{12}) = \frac{3}{4}\cos^2(\frac{\pi}{12})$ ۷

از طرفی $\longrightarrow \cos^2(\frac{\pi}{12}) = \frac{1 + \cos(\frac{\pi}{6})}{2}$ (فرمول طلایی)

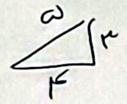
$\cos^2(\frac{\pi}{12}) = \frac{\sqrt{3}+2}{4} \longrightarrow \frac{3}{4} \times \frac{\sqrt{3}+2}{4}$

$= \frac{3\sqrt{3}+6}{16}$

$x \longrightarrow$ ③

$\frac{1-\sin x}{1+\sin x} = 4 \longrightarrow 1 - \sin x = 4 + 4\sin x$

$5\sin x = -3 \longrightarrow \sin x = -\frac{3}{5}$



$\cos x = -\frac{4}{5}$ ۸

$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x} = \frac{-\frac{3}{5}}{1-\frac{4}{5}} = \frac{-\frac{3}{5}}{\frac{1}{5}} = -3$

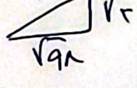
$\frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = k\cot\frac{\theta}{2}$

* $\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}$

$\frac{1}{\tan\frac{\theta}{2}} \quad \frac{1}{\tan\frac{\theta}{2}} = \cot\frac{\theta}{2} + \cot\frac{\theta}{2} = 2\cot\frac{\theta}{2} = k\cot\frac{\theta}{2}$ ۹

$k = 2$

$\alpha \longrightarrow$ ②



$\cos\alpha = \frac{-\sqrt{98}}{10}$

$\sin\alpha = \frac{\sqrt{2}}{10}$

$\cos\left(\frac{11\pi}{4} + \alpha\right) \xrightarrow{\frac{11\pi}{4} = \frac{3\pi}{4}} \cos\left(\frac{3\pi}{4} + \alpha\right) = \underbrace{\cos(\frac{3\pi}{4})}_{-\frac{\sqrt{2}}{2}}\cos\alpha - \underbrace{\sin(\frac{3\pi}{4})}_{\frac{\sqrt{2}}{2}}\sin\alpha$ 10

$= -\frac{\sqrt{2}}{2}\cos\alpha - \frac{\sqrt{2}}{2}\sin\alpha = -\frac{\sqrt{2}}{2}(\cos\alpha + \sin\alpha) = -\frac{\sqrt{2}}{2}\left(\frac{\sqrt{2}}{10} - \frac{\sqrt{98}}{10}\right) = \frac{-2}{20} + \frac{\sqrt{196}}{20} = \frac{12}{20} = 0/6$