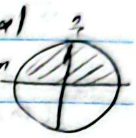
سوالات صفحه ۱۴، ۲۵

۱- $\cot_\alpha ,\ \dfrac{\cos\alpha}{\sqrt{1-\cos^2_\alpha}} ,\ \dfrac{1}{\sqrt{\cos^2_\alpha}} - \dfrac{1}{\cot_\alpha} = \dfrac{1-\sin\alpha}{|\cos\alpha|}$

انتهای کمان α :

$\dfrac{\cos_\alpha}{\sin_\alpha} = \dfrac{\cos\alpha}{\sqrt{\underbrace{1-\cos^2_\alpha}_{\Rightarrow \sin^2_\alpha}}} \xrightarrow{\div \cos_\alpha} \dfrac{1}{\sin\alpha} = \dfrac{1}{|\sin\alpha|} \Longrightarrow \sin_\alpha = |\sin\alpha|$

$\sin_\alpha > 0$



$\dfrac{1}{\sqrt{\cos^2_\alpha}} = \dfrac{1}{|\cos_\alpha|} \Longrightarrow \overbrace{\dfrac{1}{|\cos\alpha|} - \dfrac{\sin\alpha}{\cos_\alpha}}^{1-\sin_\alpha} = \dfrac{1-\sin\alpha}{\cos_\alpha} \longrightarrow \cos_\alpha > 0$

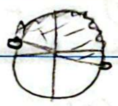
(۲)

$\sin_\alpha > 0 ,\ \cos\alpha > 0$ — α انتهای کمان در ناحیه اول است

۲- $-\dfrac{\pi}{12} < x < \dfrac{5\pi}{12}$ $\qquad \sin_{2x} = \dfrac{m-1}{4}$ $\qquad$ مجموعه مقادیر $m=?$

$m : (-1, 5]$

$\times 2$: $-\dfrac{\pi}{6} < 2x < \dfrac{5\pi}{6}$



$-\dfrac{1}{2} < \sin_{2x} \le 1$

(۲)

$-\dfrac{1}{2} < \dfrac{m-1}{4} \le 1 \longrightarrow -2 < m-1 \le 4 \longrightarrow -1 < m \le 5$

۳- $\tan_x + \cot_x = -4$ $\qquad 3\pi < 4x < 4\pi$

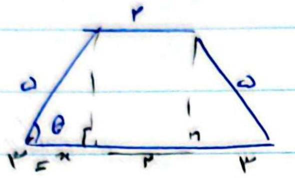
$\dfrac{1}{\sin^3_x + \cos^3_x} \overset{?}{=}$ فرمول مجموع مکعبات $\dfrac{1}{(\cos_x + \sin_x)(\underbrace{\cos^2_x + \sin^2_x}_{1} - \underbrace{\cos_x \sin_x}_{-\frac{1}{4}})} = \dfrac{1}{\frac{2}{\sqrt{2}} \times \frac{5}{4}} = \dfrac{8}{5\sqrt{2}}$ $\boxed{= \dfrac{4\sqrt{2}}{5}}$

(۱)

$\tan_x + \cot_x = \dfrac{\sin_x}{\cos_x} + \dfrac{\cos_x}{\sin_x} = \dfrac{\overbrace{\sin^2_x + \cos^2_x}^{1}}{\cos_x \sin_x} = -4 \to \cos_x \sin_x = -\dfrac{1}{4}$

$\underbrace{(\cos_x + \sin_x)^2}_{x} = \underbrace{\cos^2_x + \sin^2_x}_{1} - 2\underbrace{\sin_x \cos_x}_{-\frac{1}{4}} \Rightarrow \cos_x + \sin_x = \sqrt{\dfrac{4}{2}} = \dfrac{2}{\sqrt{2}}$

هلیا صفدریان



۴-

$\cos\theta = 0/6 \quad S = ? \rightarrow \sin\theta = ? \rightarrow 1-\cos^2\alpha = \sin^2\alpha \quad 0/36 \quad 0/64 \quad (0/8)^2$

$x = 5\cos\theta = 5\times 0/6 = 3$

$h = 5\sin\theta = 5\times 0/8 = 4$

$S = \frac{1}{2}\times 4\,(2+8) = 20$

۵-

$\tan(285^\circ)\tan(-165^\circ) - \sin(1095^\circ)\cos(255^\circ) = k\cos^2 15$

$(270+15) \quad (-180+15) \quad (\underbrace{3\times 360}_{1080}+15^\circ) \quad (270-15^\circ)$

$k = ?$ $\quad \boxed{k=-1}$

$\tan(\frac{3\pi}{2}+15)\tan(-\pi+15) - \sin(6\pi+15^\circ)\cos(\frac{3\pi}{2}-15^\circ)$

$(-\cot_{15})(\tan_{15}) - (\sin_{15})(-\sin_{15}) = -1+\sin^2_{15} = -(\cos^2_{15}) = +k\cos^2_{15} \quad -1$

$A = \sqrt{3}\cos(210^\circ)\sin(243) - \sqrt{2}\sin(135)\cos(153)$

$\quad -\frac{\sqrt{3}}{2} \qquad \frac{\sqrt{2}}{2}$

$-\frac{3}{2}\sin(270-27) - \frac{2}{2}\cos(180-27) = -\frac{3}{2}\sin(\frac{3\pi}{2}-27) - \cos$

$(\pi-27) = -\frac{3}{2}(-\cos_{27}) + \cos_{27} = \frac{3}{2}\cos_{27} + \cos_{27} = \frac{5}{2}\cos_{27}$

$\frac{\frac{5}{2}\cos_{27}}{\cos_{27}} = \boxed{\frac{5}{2}}$

۷-

$f(x) = 16\cos^2(3x)\cos^2(6x)\cos^2(12x)\cos^2(24x) \qquad f(\frac{\pi}{36}) = ?$

$\cos^2(6\times\frac{\pi}{36}) = (\frac{\sqrt{3}}{2})^2 = \frac{3}{4}$

$\cos^2(12x) = \cos^2(12\times\frac{\pi}{36}) = \frac{1}{4}$

$\cos^2(3x) = \cos^2(\frac{\pi}{12}) \Rightarrow \frac{1+\frac{\sqrt{3}}{2}}{2}$

$\cos^2\frac{\pi}{12} = \sqrt{\frac{1+\cos\frac{\pi}{6}}{2}} = \frac{|1+\sqrt{3}|}{2} = \frac{1+\sqrt{3}}{2}$

$\cos^2(24x) = \cos^2(24\times\frac{\pi}{36}) = \frac{1}{4}$

$= 16\times\frac{3}{4}\times\frac{1}{4}\times\frac{1}{4}\times\frac{1+\sqrt{3}}{2} = \frac{3+3\sqrt{3}}{8}$

حل معادلات

۸- در ربع ۳ — ۱٫۷۵

$$\frac{1-\sin\alpha}{1+\sin\alpha}=4 \rightarrow 1-\sin\alpha = 4+4\sin\alpha$$

$$-3 = 5\sin\alpha \rightarrow \sin\alpha = -\frac{3}{5}$$

$\tan\frac{\alpha}{2}=?$

$$\tan\frac{\alpha}{2}+\cot\frac{\alpha}{2}=\frac{2}{\sin(2\cdot\frac{\alpha}{2})}=-\frac{10}{3} \rightarrow \tan\frac{\alpha}{2}+\frac{1}{\tan\frac{\alpha}{2}}=-\frac{10}{3}$$

$\left(\sin\alpha=-\frac{3}{5}\right)$

$$3\left(\tan\frac{\alpha}{2}\right)^2+10\tan\frac{\alpha}{2}+3=0 \rightarrow \left(\tan\frac{\alpha}{2}+3\right)\left(3\tan\frac{\alpha}{2}+1\right) \rightarrow \tan\frac{\alpha}{2}=-3 \;\checkmark, \quad \tan\frac{\alpha}{2}=-\frac{1}{3} \;\times$$

۹- ۱

$$\frac{\sin\theta}{1-\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=x\cot\frac{\theta}{2} \qquad \boxed{x=1}$$

$$\frac{\sin\theta(1+\cos\theta)}{\sin^2\theta} \rightarrow \frac{\sin(1+\cos\theta)}{\sin^2\theta}=\frac{2\sin(1+\cos\theta)}{2\sin^2\theta}=\frac{1+\cos\theta}{\sin\theta}$$

طرق

$$\frac{\sin\alpha}{1+\cos\alpha}=\frac{1-\cos\theta}{\sin\alpha}=\tan\frac{\alpha}{2} \rightarrow \frac{1+\cos\theta}{\sin\theta}=\cot\frac{\theta}{2}$$

۱۰- ۰

$$\sin\alpha=\frac{\sqrt{2}}{10} \qquad \cos\left(\frac{11\pi}{4}+\alpha\right)=? \quad = \cos\left(\frac{3\pi}{4}+\alpha\right) = -\sin\alpha = \frac{+\sqrt{2}}{10}$$

۳) $\frac{\sin^2 x+\cos^2 x}{\sin x\cos x} = -3 \to \sin x\cos x = \frac{-1}{3} = A$

$\frac{1}{\sin^3 x+\cos^3 x} = \frac{1}{(\sin x+\cos x)(1-\sin x\cos x)}$

$A^2 = \sin^2 x+\cos^2 x+2\sin x\cos x = \frac{1}{3}$

$\to A \begin{cases} \frac{1}{\sqrt{3}} & \times \\ -\frac{1}{\sqrt{3}} & \checkmark \end{cases}$

$\hookrightarrow \frac{-9}{4\sqrt{3}} = -0.75\sqrt{3}$

۷) $f\left(\frac{\pi}{36}\right) = 16\cos^2\left(\frac{\pi}{12}\right)\cos^2\left(\frac{\pi}{6}\right)\cos^2\left(\frac{\pi}{3}\right)\cos^2\left(\frac{2\pi}{3}\right)$

$\cos^2\frac{\pi}{12} = \frac{1+\cos\frac{\pi}{6}}{2} = \frac{2+\sqrt{3}}{4}$

$\downarrow$

$16\left(\frac{2+\sqrt{3}}{4}\right)\times\frac{3}{4}\times\frac{1}{4}\times\frac{1}{4}$

$= \frac{3(2+\sqrt{3})}{16}$

۹) $\frac{\sin^2\theta+(1-\cos^2\theta)}{(1-\cos\theta)\sin\theta} = \frac{2\sin^2\theta}{\sin\theta(1-\cos\theta)} = \frac{2\times 2\times\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} = 2\cot\frac{\theta}{2}$

$\to K = 2$

۱۰) $\cos\left(\frac{11\pi}{4}+\alpha\right) = -\left(\cos\alpha\cos\frac{\pi}{4}+\sin\alpha\sin\frac{\pi}{4}\right)$

$\to \frac{-\sqrt{2}}{2}(\cos\alpha+\sin\alpha)$ $\qquad \cos\alpha = \frac{-7\sqrt{2}}{10}$

$\hookrightarrow -\frac{\sqrt{2}}{2}\left(\frac{-7\sqrt{2}}{10}+\frac{\sqrt{2}}{10}\right) = \frac{3}{5}$