حل ۱ صفحه ۵۰

۱- $\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}}$, $\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|}$

انتهای کمان α :

$$\frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{\sqrt{\underbrace{1-\cos^2\alpha}_{\Rightarrow \sin^2\alpha}}} \xrightarrow{\div \cos\alpha} \frac{1}{\sin\alpha} = \frac{1}{|\sin\alpha|} \Longrightarrow \sin\alpha = |\sin\alpha| \quad \sin\alpha > 0$$

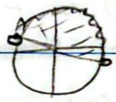
$$\frac{1}{\sqrt{\cos^2\alpha}} = \frac{1}{|\cos\alpha|} \Rightarrow \overbrace{\frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha}}^{1-\sin\alpha} = \frac{1-\sin\alpha}{\cos\alpha} \Rightarrow \cos\alpha > 0$$

$\sin\alpha > 0$, $\cos\alpha > 0$ $\qquad$ α ← انتهای کمان در ناحیهٔ اول است

۲- $-\frac{\pi}{12} < x < \frac{5\pi}{12}$ $\qquad$ $\sin 2x = \frac{m-1}{4}$ $\qquad$ مجموعه مقادیر m = ?

$m : (-1, 5]$

$\times 2$: $-\frac{\pi}{6} < 2x < \frac{5\pi}{6}$



$-\frac{1}{2} < \sin 2x \le 1$

$$-\frac{1}{2} < \frac{m-1}{4} \le 1 \to -2 < m-1 \le 4 \to -1 < m \le 5$$

۳- $\tan x + \cot x = -3$ $\qquad$ $3\pi < 4x < 4\pi$

$$\frac{1}{\sin^3 x + \cos^3 x} = ? \quad \text{(فرمول دو مکعب)} \quad \frac{1}{(\cos x + \sin x)(\underbrace{\cos^2 x + \sin^2 x}_{1} - \underbrace{\cos x \sin x}_{-\frac{1}{3}})} = \frac{1}{\frac{2}{\sqrt3}\times\frac{4}{3}} = \frac{8}{3\sqrt3}$$

$$= \frac{3\sqrt3}{8}$$

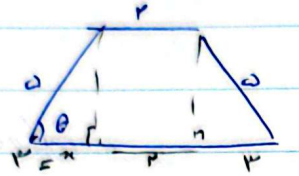
$$\tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\overbrace{\sin^2 x + \cos^2 x}^{1}}{\cos x \sin x} = -3 \to \cos x \sin x = -\frac{1}{3}$$

$$(\cos x + \sin x)^2 = \underbrace{\cos^2 x + \sin^2 x}_{1} - 2\underbrace{\sin x \cos x}_{-\frac{1}{3}} \Rightarrow \cos x + \sin x = -\sqrt{\frac{4}{3}} = -\frac{2}{\sqrt3}$$

حل تمرین

$\cos\theta = \frac{4}{5}$ → $\sin\theta = ?$ → $1-\cos^2\alpha = \sin^2\alpha$ (۴/۵)

$S = ?$



$x = a\cos\theta = 5 \times \frac{4}{5} = 4$

$h = a\sin\theta = 5 \times \frac{3}{5} = 3$

$S = \frac{1}{2} \times 3 \times (2+8) = 15$

۵- $\tan(\frac{3\pi}{2}+\alpha)\tan(-\pi+\alpha) - \sin(\frac{5\pi}{2}+\alpha)\cos(2\pi-\alpha) = k\cos^2\alpha$

$k = ?$

$k = -1$

$\tan(\frac{11\pi}{2}+\alpha)\tan(-\pi+\alpha) - \sin(6\pi+\alpha)\cos(\frac{3\pi}{2}-\alpha)$

$(-\cot\alpha)(\tan\alpha) - (\sin\alpha)(-\sin\alpha) = -1 + \sin^2\alpha = -(\cos^2\alpha) = -k\cos^2\alpha$

۶- $A = \sqrt{2}\cos(\frac{\pi}{4}) \sin(\frac{5\pi}{4}) - \sqrt{2}\sin(\frac{\pi}{4})\cos(\frac{5\pi}{4})$

$-\frac{\sqrt{2}}{2}$ $\frac{\sqrt{2}}{2}$

$\cos(\pi+\alpha) = -\cos\alpha$ $\sin(\pi+\alpha) = -\sin\alpha$

$-\frac{3}{2}\sin(270-\alpha) - \frac{1}{2}\cos(180-\alpha) = -\frac{3}{2}\sin(\frac{3\pi}{2}-\alpha) - \cos(\pi-\alpha)$

$= -\frac{3}{2}(-\cos\alpha) + \cos\alpha = \frac{3}{2}\cos\alpha + \cos\alpha = \frac{5}{2}\cos\alpha$

$\frac{\frac{5}{2}\cos\alpha}{\cos\alpha} = \frac{5}{2}$

۷-

$f(x) = 16\cos^2(3x)\cos^2(9x)\cos^2(12x)\cos^2(24x)$ $f(\frac{\pi}{24}) = ?$

$\cos^2(9\times\frac{\pi}{24}) = (\frac{\sqrt{2}}{2})^2 = \frac{1}{2}$

$\cos^2(12x) = \cos^2(12\times\frac{\pi}{24}) = \frac{1}{2}$

$\cos^2(3x) = \cos^2(\frac{\pi}{8}) \Rightarrow \frac{1+\sqrt{2}}{2}$

ضلع ضلع

$\cos^2\frac{\pi}{8} = \sqrt{\frac{1+\cos\frac{\pi}{4}}{2}}^2 = \frac{|1+\sqrt{2}|}{2} = \frac{1+\sqrt{2}}{2}$

$\cos^2(24x) = \cos^2(24\times\frac{\pi}{24}) = \frac{1}{2}$

$= 16 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1+\sqrt{2}}{2} = \frac{2+2\sqrt{2}}{8}$

حل تمرین ۱

$\frac{1-\sin\alpha}{1+\sin\alpha} = 4 \rightarrow 1-\sin\alpha = 4+4\sin\alpha$

$-3 = 5\sin\alpha \rightarrow \sin\alpha = -\frac{3}{5}$

$\tan\frac{\alpha}{2}$ ؟

$\tan\frac{\alpha}{2} + \cot\frac{\alpha}{2} = \frac{2}{\sin(2\alpha\cdot\frac{\alpha}{2})} = \frac{2}{\sin\alpha} = -\frac{10}{3}$ $\rightarrow \tan\frac{\alpha}{2} + \frac{1}{\tan\frac{\alpha}{2}} = -\frac{10}{3}$

$3(\tan\frac{\alpha}{2})^2 + 10\tan\frac{\alpha}{2} + 3 = 0 \rightarrow (\tan\frac{\alpha}{2}+3)(3\tan\frac{\alpha}{2}+1)$ $\rightarrow \tan\frac{\alpha}{2} = -3$

$\rightarrow \tan\frac{\alpha}{2} = -\frac{1}{3}$ ۹

$\frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = k\cot\frac{\theta}{2}$ $\quad k=1$

$\frac{\sin\theta(1+\cos\theta)}{\sin^2\theta} \rightarrow \frac{\sin(1+\cos\theta)}{\sin^2\theta} = \frac{2\sin(1+\cos\theta)}{2\sin^2\theta} = \frac{1+\cos\theta}{\sin\theta}$

ثابت $\frac{\sin\alpha}{1+\cos\alpha} + \frac{1-\cos\theta}{\sin\alpha} = \tan\frac{\alpha}{2} \rightarrow \frac{1+\cos\theta}{\sin\theta} = \cot\frac{\theta}{2}$

۶

$\sin\alpha = \frac{\sqrt{3}}{5}$ $\quad \cos(\frac{11\pi}{4}-\alpha)$ ؟ $= \cos(\frac{4\pi}{4}+\frac{3\pi}{4}-\alpha) = -\sin\alpha = -\frac{\sqrt{3}}{5}$