شهرزاد شوشتری

۱۲,۵

۱)

$\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} = \frac{\cos\alpha}{|\sin\alpha|}$, $\frac{1}{|\cos\alpha|} - \frac{|\sin\alpha|}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \rightarrow \sin\alpha \rightarrow +$, $\cos\alpha \rightarrow +$

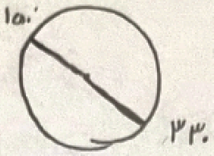
ربع اول

۲

---

۲)

$\sin 2x = \frac{m-1}{3} \rightarrow -\frac{\pi}{4} < 2x < \frac{5\pi}{4}$



$\rightarrow -\frac{1}{2} < \frac{m-1}{3} < \frac{1}{2} \rightarrow -2 < m-1 < 2$

$-1 < m < 3$

۱

$\frac{\pi}{8} < x < \pi$

$-\frac{1}{2} < \sin 2x \leq 1 \rightarrow -\frac{1}{2} < \frac{m-1}{4} \leq 1 \rightarrow m \in (-1, 5]$

---

۳)

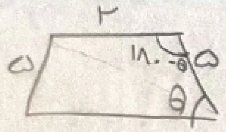
$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = -3$ $\quad \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{\frac{1}{2}\sin 2x} \rightarrow \frac{2}{\sin 2x} = -3 \rightarrow -3\sin 2x = 2 \rightarrow \sin 2x = \frac{-2}{3}$

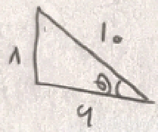
$2\sin x \cos x = \frac{-2}{3} \rightarrow = -\frac{1}{3}$

۰

---

۴)





$\sin(180-\theta) = \sin\theta = \frac{8}{10} = \frac{4}{5} \rightarrow S = \frac{4}{5} \times 2 \times 5 \times \frac{1}{2}$ $\sqrt{4}$

$4 \times 2 = 8$

$\cos\theta = \frac{3}{5} = \frac{6}{10}$

$\sin\theta = \frac{4}{5} = \frac{8}{10}$

$S = \frac{(2+8)}{2} \times 4 = 20$

۰,۵

---

۵)

$\tan(270+15) \times (\tan(-180+15)) - \sin(360+15) \times \cos(270-15)$

$-\cot 15 \times \tan 15 - \sin 15 \times -\sin 15 = -1 + \sin^2 15 \rightarrow$ ~~$-\sin^2 15 + \cos^2 15 + \sin^2 15$~~ $= -\cos^2 15$

$\tan = \frac{1}{\cot}$

۲

---

۶)

$\sqrt{3} \times -\frac{\sqrt{3}}{2} \times \sin(270-27) - \sqrt{2} \times \frac{\sqrt{2}}{2} \times \cos(180-27)$

$-\frac{3}{2} \times -\cos 27 + \cos 27 = \frac{3}{2}\cos 27 + \cos 27 = \frac{5}{2}\cos 27$ $\quad \frac{5}{2}$

۲

شهرزاد شوشتری

۷)

$\frac{1}{2}\sin^2(6x)$, $\frac{1}{2}\sin^2(12x)$, $\frac{1}{2}\sin^2(24x)$, $\frac{1}{2}\sin^2(48x)$

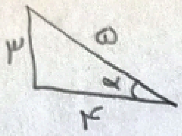
$$f(x) = \frac{16\cos^2(3x)\cos^2(6x)\cos^2(12x)\cos^2(24x) \times \sin^2(3x)}{\sin^2(3x)} =$$

۰/۵

$$\frac{16 \times \frac{1}{16} \times \sin^2(48x)}{\sin^2(3x)} = \frac{\sin^2(\frac{4\pi}{3})}{\sin^2(\frac{\pi}{12})}$$

۸)

$$4 + 4\sin x = 1 - \sin x \rightarrow 5\sin x = -3 \rightarrow \sin x = -\frac{3}{5}$$



$$\cos x = -\frac{4}{5} \qquad \tan\frac{\alpha}{2} = \frac{1+\frac{4}{5}}{-\frac{3}{5}} = \frac{\frac{9}{5}}{-\frac{3}{5}} = -\frac{9}{3} = \boxed{-3}$$

۱

۹)

$$\frac{\sin\theta}{1-\cos\theta} = \cot\frac{\alpha}{2} \qquad \frac{1+\cos\theta}{\sin\theta} = \cot\frac{\theta}{2} \rightarrow 2\cot\frac{\theta}{2} \rightarrow \boxed{12{,}2}$$

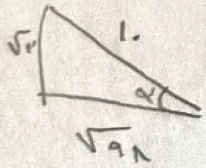
۱

۱۰)

$$\cos(\frac{11\pi}{4}+\alpha) = \cos\frac{11\pi}{4} \times \cos\alpha - \sin\frac{11\pi}{4} \times \sin\alpha = -\frac{\sqrt{2}}{2} \times -\frac{\sqrt{2}}{10} - \frac{\sqrt{2}}{2} \times \frac{7\sqrt{2}}{10} =$$

$$= \frac{1}{10} - \frac{7}{10} = -\frac{6}{10} = -\frac{3}{5}$$

۰/۵



$$\sin\alpha = \frac{\sqrt{98}}{10} \qquad \cos\alpha = -\frac{\sqrt{2}}{10}$$

$$\cos\frac{11\pi}{4} = \cos\frac{3\pi}{4}$$

$$\sin\frac{11\pi}{4} = \sin\frac{3\pi}{4}$$

$$\sqrt{98} = 7\sqrt{2}$$

$$\cos(\frac{11\pi}{4}+\alpha) = -(\cos\alpha\cos\frac{\pi}{4} + \sin\alpha\sin\frac{\pi}{4})$$

$$\rightarrow -\frac{\sqrt{2}}{2}(\cos\alpha + \sin\alpha) \qquad \cos\alpha = \frac{-7\sqrt{2}}{10}$$

$$\hookrightarrow -\frac{\sqrt{2}}{2}\left(\frac{-7\sqrt{2}}{10} + \frac{\sqrt{2}}{10}\right) = \frac{3}{5}$$

۳) $\dfrac{\sin^۳ x+\cos^۳ x}{\sin x\cos x} = -۳ \longrightarrow \sin x\cos x = \dfrac{-۱}{۳} = A$

$\dfrac{۱}{\sin^۳ x+\cos^۳ x} = \dfrac{۱}{(\sin x+\cos x)(۱-\sin x\cos x)}$

$A^۲ = \sin^۲ + \cos^۲ x + ۲\sin x\cos x = \dfrac{۱}{۳}$

$\rightarrow A \begin{cases} \dfrac{۱}{\sqrt{۳}} \ \times \\ -\dfrac{۱}{\sqrt{۳}} \ \checkmark \end{cases}$

$\rightarrow \dfrac{-۹}{۴\sqrt{۳}} = -۰/۷۵\sqrt{۳}$

۷) $f\left(\dfrac{\pi}{۳۶}\right) = ۱۶\cos^۲\left(\dfrac{\pi}{۱۲}\right)\cos^۲\left(\dfrac{\pi}{۶}\right)\cos^۲\left(\dfrac{\pi}{۳}\right)\cos^۲\left(\dfrac{۲\pi}{۳}\right)$

$\downarrow$

$۱۶\left(\dfrac{۲+\sqrt{۳}}{۴}\right) \times \dfrac{۳}{۴} \times \dfrac{۱}{۴} \times \dfrac{۱}{۴}$

$= \dfrac{۳(۲+\sqrt{۳})}{۱۶}$

$\cos^۲\dfrac{\pi}{۱۲} = \dfrac{۱+\cos\dfrac{\pi}{۶}}{۲} = \dfrac{۲+\sqrt{۳}}{۴}$