گروه یازدهم دختر C   ۱۹   دینا جمشیدی   تکلیف شماره ۲۸

۱) ناحیه اول

$$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|}$$

$$\sqrt{\cos^2\alpha} = |\cos\alpha|$$

$$\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} \Rightarrow \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{|\sin\alpha|}$$

$$\sqrt{\sin^2\alpha} = |\sin\alpha|$$

$$|\sin\alpha| = \sin\alpha \to \boxed{\sin\alpha \geq 0}$$

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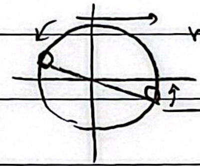
$$\frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{|\cos\alpha|} \to$$

$$|\cos\alpha| = \cos\alpha \to \boxed{\cos\alpha \geq 0}$$

۲)

$$-\frac{\pi}{12} < x < \frac{5\pi}{12} \qquad \sin 2x = \frac{m-1}{4}$$

$$-\frac{\pi}{6} < 2x < \frac{5\pi}{6}$$



$\max \sin\alpha = 1$

$\min \sin\alpha = -\frac{1}{2}$

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$$-\frac{1}{2} < \sin 2x \leq 1 \qquad -\frac{1}{2} < \frac{m-1}{4} \leq 1$$

$$-2 < m-1 \leq 4$$

$$-1 < m \leq 5 \qquad m \in (-1, 5]$$

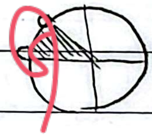
۳)

$$\tan\alpha + \cot\alpha = -3, \quad 3\pi < 4x < 4\pi \qquad \frac{1}{\sin^3 x + \cos^3 x} \ ?$$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = -3 = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = -3$$

$$\sin x \cos x = -\frac{1}{3} \qquad (\sin x + \cos x)^2 = \frac{1}{3}$$

$$\underbrace{\sin^2 x + \cos^2 x}_{1} + \underbrace{2\sin x \cos x}_{-\frac{2}{3}} = \frac{1}{3}$$



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$$\sin x + \cos x = \pm\frac{1}{\sqrt{3}} \qquad \frac{3\pi}{4} < x < \pi$$

$$\sin x + \cos x < 0 \to = -\frac{1}{\sqrt{3}}$$

$$\frac{1}{\sin^3 x + \cos^3 x} \to \underbrace{(\sin x + \cos x)^3}_{-\frac{1}{3\sqrt{3}}} - \overbrace{3\underbrace{(\sin x \cos x)}_{-\frac{1}{3}}\underbrace{(\sin x + \cos x)}_{-\frac{1}{\sqrt{3}}}}^{\frac{1}{\sqrt{3}}}$$

$$-\frac{1}{3\sqrt{3}} - \frac{1}{\sqrt{3}} = \frac{-1-3}{3\sqrt{3}} = \frac{-4}{3\sqrt{3}}$$

معکوس

$$\frac{1}{\sin^3 x + \cos^3 x} = \frac{-3\sqrt{3}}{4}$$

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$$f(x) = ۱۶\cos^۲(۳x)\cos^۲(۶x)\cos^۲(۱۲x)\cos^۲(۲۴x) \quad (۷)$$

$$f\left(\frac{\pi}{۳۶}\right) - ۹ = ۱۶\cos^۲\left(\frac{\pi}{۱۲}\right)\underbrace{\cancel{\cos^۲\left(\frac{\pi}{۶}\right)}}_{\frac{۳}{۴}}\underbrace{\cancel{\cos^۲\left(\frac{\pi}{۳}\right)}}_{\frac{۱}{۴}}\underbrace{\cancel{\cos^۲\left(\frac{۲\pi}{۳}\right)}}_{\frac{۱}{۴}}$$

$$= \cancel{۱۶} \times \frac{۳}{۴} \times \frac{۱}{۱۶} \times \cos^۲\frac{\pi}{۱۲}$$

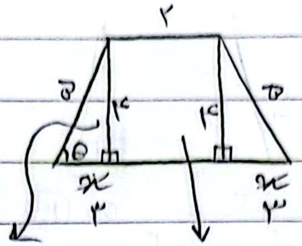
$$= \frac{۳}{۴}\cos^۲\left(\frac{\pi}{۱۲}\right)$$

$$\cos^۲\alpha = \frac{۱+\cos ۲\alpha}{۲} \to \cos^۲\left(\frac{\pi}{۱۲}\right) = \frac{۱+\frac{\sqrt{۳}}{۲}}{۲}$$

$$\cos^۲\left(\frac{\pi}{۱۲}\right) = \frac{۱}{۲} + \frac{\sqrt{۳}}{۴} = \frac{۲+\sqrt{۳}}{۴}$$

$$\frac{۳}{۴} \times \frac{۲+\sqrt{۳}}{۴} = \frac{۶+۳\sqrt{۳}}{۱۶}$$

(۴)



$$\cos\theta = \frac{x}{۵} = ۰/۶$$

$$x = ۳$$

$$S = \frac{۳\times۴}{۲} = ۶ \qquad S = ۸$$

$$S_{کل} = ۲(۶) + ۸ = ۲۰$$

$$\tan(۲۸۵^\circ)\tan(-۱۶۵^\circ) - \sin(۱۸۵^\circ)\cos(۲۵۵^\circ) \quad (۵)$$

$$= K\cos^۲ ۱۵^\circ \qquad K = ?$$

$$\underbrace{\tan\left(\frac{۳\pi}{۲} + ۱۵^\circ\right)}_{-\cot ۱۵^\circ}\ \underbrace{\tan(۱۵^\circ - \pi)}_{+\tan ۱۵^\circ} \Rightarrow -۱$$

$$-\sin(۱۵^\circ)\underbrace{\cos\left(\frac{۳\pi}{۲} - ۱۵^\circ\right)}_{-\sin ۱۵^\circ} =$$

$$= -۱ + \sin^۲ ۱۵^\circ = -\cos^۲ ۱۵^\circ \to K = -۱$$

$$A = \sqrt{۳}\cos(۲۱۰^\circ)\sin(۲۴۳^\circ) \quad (۶)$$

$$-\sqrt{۲}\sin(۱۳۵^\circ)\cos(۱۵۳^\circ) =$$

$$\sqrt{۳}\left(-\frac{\sqrt{۳}}{۲}\right)\underbrace{\sin\left(\frac{۳\pi}{۲} - ۲۷^\circ\right)}_{-\cos ۲۷^\circ} - \underbrace{\sqrt{۲}\left(+\frac{\sqrt{۲}}{۲}\right)}_{-۱}\underbrace{\cos(\pi - ۲۷^\circ)}_{-\cos ۲۷^\circ}$$

$$\frac{۳}{۲}\cos ۲۷^\circ + \cos ۲۷^\circ = \frac{۵}{۲}\cos ۲۷^\circ$$

$\frac{۵}{۲}$ برابر

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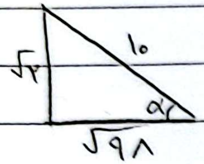
۸

۱۰

$\sin\alpha = \frac{\sqrt{۲}}{۱۰}$ $\qquad \cos\left(\frac{11\pi}{4}+\alpha\right)$

$\cos\alpha = \frac{-۷\sqrt{۲}}{۱۰}$ $\qquad \frac{۷\sqrt{۲}}{۱۰}$



$\sqrt{۲}\sin\left(\alpha+\frac{\pi}{4}\right) = \sin\alpha + \cos\alpha$

$\sin\left(\alpha+\frac{\pi}{4}\right) = \frac{-۳}{۵} \rightarrow \cos\left(\alpha+\frac{\pi}{4}\right) = -\frac{۴}{۵}$

$|\sin\alpha| < |\cos\alpha| \rightarrow \frac{۳\pi}{4} < \alpha < \pi \rightarrow$

$\pi < \alpha + \frac{\pi}{4} < \frac{۵\pi}{4} \rightarrow \cos\left(\alpha+\frac{\pi}{4}\right) < 0$

$\cos\left(\frac{11\pi}{4}+\alpha\right) = \cos\left(\pi - \left(\frac{\pi}{4}+\alpha\right)\right) =$

$\frac{۳\pi}{4}$ $\qquad \underbrace{-\cos\left(\alpha+\frac{\pi}{4}\right)}_{-\frac{۴}{۵}} = +\frac{۴}{۵}$

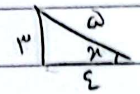
$\cos\left(\frac{11\pi}{4}+\alpha\right) = -\left(\cos\alpha\cos\frac{\pi}{4} + \sin\alpha\sin\frac{\pi}{4}\right)$

$\rightarrow \frac{-\sqrt{۲}}{۲}(\cos\alpha + \sin\alpha)$ $\qquad \cos\alpha = \frac{-۷\sqrt{۲}}{۱۰}$

$\hookrightarrow \frac{-\sqrt{۲}}{۲}\left(\frac{-۷\sqrt{۲}}{۱۰} + \frac{\sqrt{۲}}{۱۰}\right) = \frac{۳}{۵}$

$\frac{1-\sin x}{1+\sin x} = ۴$ $\qquad ۴ + ۴\sin x = 1 - \sin x$

$۵\sin x = -۳$ $\qquad \sin x = -\frac{۳}{۵}$



$\cos x = -\frac{۴}{۵}$

$\tan\frac{x}{۲} = \frac{\sin x}{1+\cos x} = \frac{-\frac{۳}{۵}}{1-\frac{۴}{۵}} = -۳$

۹

$\frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = k\cot\frac{\theta}{۲}$?

$\frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha} = \tan\frac{\alpha}{۲}$

$\cot\frac{\theta}{۲} + \cot\frac{\theta}{۲} = ۲\cot\frac{\theta}{۲} \rightarrow k = ۲$

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