۱

$$\frac{1-\sin\alpha}{|\cos\alpha|} \to \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \to \frac{\sin\alpha}{\cos\alpha} = \frac{1-1+\sin\alpha}{|\cos\alpha|}$$

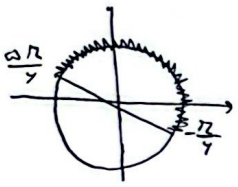
$$\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} = \frac{\cos\alpha}{|\sin\alpha|} \to \frac{\cos\alpha}{|\sin\alpha|} = \frac{\cos\alpha}{\sin\alpha} \to \sin\alpha \geq 0$$

[2]

$\cos\alpha \geq 0$ [1]

ناحیه اول

۲



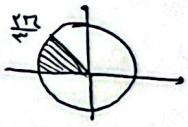
$$-\frac{\pi}{4} < x < \frac{5\pi}{4} \leftarrow -\frac{\pi}{4} < x < \frac{5\pi}{4}$$

$$-\frac{1}{2} < \frac{m-1}{4} \leq 1 \to -2 < m-1 \leq 4 \to \boxed{-1 < m \leq 5}$$

۳

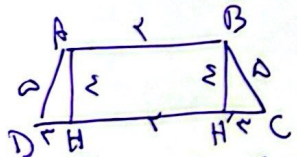
$$\frac{1}{\cos^2 x + \sin^2 x} = \frac{1}{(\sin+\cos)(\sin^2 - \sin\cos+\cos^2)} = \frac{1}{\frac{3}{2}(\sin+\cos)} \quad (\sin x+\cos x)^2 = 1 + 2\sin x\cos x \quad (-\tfrac{1}{2})$$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = -2 \to \frac{\sin^2 x + \cos^2 x}{\sin x\cos x} = -2 \to \sin x\cos x = -\frac{1}{2}$$



$$\frac{1}{\frac{\sqrt{2}}{2}(\sin+\cos)} = \frac{1}{\frac{\sqrt{2}}{2}\times -\frac{1}{\sqrt{2}}} = \frac{-2\sqrt{2}}{2} = \boxed{-0.78\sqrt{2}}$$

۴



$$\cos\alpha = \frac{6}{10} \to \frac{6}{x} = \frac{DA}{AD} \to DA = 2 \to \hat{ADH} = \hat{BHC}$$

$$BH' = 2 \quad AB = HH' = 2$$

$$\hat{ADH} \to AH^2 + DH^2 = AD^2 \quad (8,\ 6,\ 10)$$

$$AH = 8 \to H'C = 4 \qquad DC = 2+2+4 = 8 \qquad S = 8\times\frac{(10)}{2} = 20$$

۵

$$\tan(225^\circ)\underbrace{\tan(-140^\circ)}_{-\tan(140)} - \sin\underbrace{(1950^\circ)}_{5\times360+150}\cos(240) = \tan(270+30)(-\tan(180-150)) - (\sin 150)\cos(270-150)$$

$$(-\cot 30)(\tan 30) - (\sin 30)(-\sin(-30)) = -1 + \sin^2 30 = \boxed{-\cos^2(30^\circ)} \to \boxed{k=1}$$

۶

$A = \sqrt{2}\cos(110^\circ)\sin(342^\circ) + \sqrt{2}\cos(342^\circ)\cos(150^\circ)$

$\cos(110^\circ) = -\cos(70^\circ) = -\cos(72^\circ)$, $\sin(342^\circ) = -\cos(72^\circ)$, $\cos(342^\circ) = \frac{\sqrt{5}+1}{4}$, $\cos(150^\circ) = -\frac{\sqrt{3}}{2}$

$A = 2\cos^2(72^\circ) - \frac{\sqrt{3}}{2}\left(\frac{\sqrt{5}+1}{4}\right) = 2\cos^2(72^\circ) - \frac{\sqrt{15}+\sqrt{3}}{8}$

$\cos(72^\circ) = \sin(18^\circ)$

$\cos(36^\circ) = 1 - 2\sin^2(18^\circ) \Rightarrow \frac{\sqrt{5}+1}{4} = 1 - 2\sin^2(18^\circ) \Rightarrow \sin^2(18^\circ) = \frac{3-\sqrt{5}}{8}$

$A = 2\left(\frac{3-\sqrt{5}}{8}\right) - \frac{\sqrt{15}+\sqrt{3}}{8} = \frac{6-2\sqrt{5}-\sqrt{15}-\sqrt{3}}{8}$

۷

$f\left(\frac{\pi}{24}\right) = 16\cos^2\frac{\pi}{24}\cos^2\frac{\pi}{12}\cos^2\frac{\pi}{6}\cos^2\frac{\pi}{3} = 16\left(\frac{1+\cos\frac{\pi}{12}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)^2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2$

$= 1 + \frac{\sqrt{3}}{2} + 1 = 2 + \frac{\sqrt{3}}{2} = \frac{4+\sqrt{3}}{2}$

۸

$\frac{1-\sin x}{1+\sin x} = 3 \rightarrow 3 + 3\sin x = 1 - \sin x$

$5\sin x = -4 \rightarrow \sin x = -\frac{4}{5}$, $\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = -\frac{4}{5} \rightarrow$

$4\tan^2\frac{x}{2} + 1 - \tan\frac{x}{2} + 4 = 0 \rightarrow (4\tan\frac{x}{2} + 1)(\tan\frac{x}{2} + 4) = 0$ → $\tan\frac{x}{2} = -\frac{1}{4}$, $\tan\frac{x}{2} = -4$

۹

$\frac{\sin\alpha}{1-\cos\alpha} = \cot\frac{\alpha}{2}$, $\frac{\sin\alpha}{1-\cos\alpha} + \frac{1+\cos\alpha}{\sin\alpha} = 2\cot\frac{\alpha}{2}$ → $k=2$

$\frac{1+\cos\alpha}{\sin\alpha} = \cot\frac{\alpha}{2}$

۱۰

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \rightarrow +\frac{\sqrt{192}}{20} - \frac{\sqrt{4}}{20} \rightarrow +\frac{12}{20} - \frac{2}{20} = \frac{+12}{20} = \pm\frac{4}{5}$

$\beta = \frac{11\pi}{4} = 2\pi + \frac{3\pi}{4}$ → $\sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$, $\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$

$\alpha \rightarrow \sin\alpha = \frac{\sqrt{2}}{10}$, $\cos\alpha = -\frac{\sqrt{98}}{10}$

$\sin^2\alpha + \cos^2\alpha = 1$

$\frac{2}{100} + \frac{98}{100}$