دریا ریالی درستی

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$\cot a = \frac{\cos a}{\sqrt{1-\cos^2 a}}$, $\frac{1}{\sqrt{\cos^2 a}} - \frac{1}{\cot a} = \frac{1-\sin a}{|\cos a|}$

$\frac{\cos a}{\sin a} = \frac{\cos a}{\sqrt{\sin^2 a}} = \frac{\cos a}{|\sin a|} = \frac{\cos a}{\sin a} \longrightarrow |\sin a| = \sin a \to \sin a > 0$

$\frac{1}{\sqrt{\cos^2 a}} - \frac{1}{\frac{\cos}{\sin}} \Rightarrow \frac{1}{|\cos a|} - \frac{\sin}{\cos} = \frac{1-\sin a}{|\cos a|} \to \cos a > 0$ → انتهای کمان در ناحیه اول مثلثاتی

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$-\frac{\pi}{12} < x < \frac{5\pi}{12}$, $\sin 2x = \frac{m-1}{4}$

$-\frac{\pi}{6} < 2x < \frac{5\pi}{6} \to -\frac{1}{2} < \sin 2x \le 1 \to \left(-\frac{1}{2} < \frac{m-1}{4} \le 1\right) \times 4$

$-2 < m-1 \le 4 \longrightarrow \boxed{-1 < m \le 5}$

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$\tan + \cot x = -3 \to \frac{1}{\sin x \cos x} = -3 \to -3\sin x \cos x = 1 \to \sin x \cos x = \frac{-1}{3}$

$\frac{3\pi}{2} < 2x < 2\pi$

$\frac{3\pi}{4} < x < \pi$; $\cos x < 0$, $\sin x > 0$.

$\frac{1}{\sin^3 x + \cos^3 x} = \frac{1}{(\sin + \cos x)} \times \frac{1}{(\sin^2 + \cos^2 - \sin\cos x)}$

$(\sin + \cos x)^2 = \underbrace{\sin^2 + \cos^2}_{1} + 2\sin x \cos x = 1 + 2 \times \frac{-1}{3} = 1 - \frac{2}{3} = \frac{1}{3}$

(با توجه به محل قرارگیری x حاصل جمع منفی می‌شود) $\sin + \cos x = -\sqrt{\frac{1}{3}} = -\frac{\sqrt{3}}{3}$

$\frac{1}{(\sin + \cos x)} \times \frac{1}{\sin^2 + \cos^2 - \sin\cos} = \frac{1}{-\sqrt{\frac{1}{3}}} \times \frac{1}{1 + \frac{1}{3}} = \frac{1}{-\frac{4}{3} \times \frac{\sqrt{3}}{3}} = \frac{1}{-\frac{4\sqrt{3}}{9}}$

$= \boxed{\frac{-9}{4\sqrt{3}}}$

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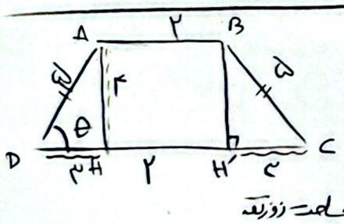

$\cos\theta = \frac{DH}{AB} \Rightarrow \frac{6}{10} = \frac{DH}{5} \to DH = 3 = H'C$

$\sqrt{5^2 - 3^2} = 4 = AH = BH'$ قاعده بزرگ $= 3 + 3 + 2 = 8$

مساحت ذوزنقه $= \frac{(\text{قاعده بزرگ} + \text{قاعده کوچک}) \times \text{ارتفاع}}{2} = \frac{10 \times 4}{2} = 20$

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$\tan(285)\tan(-165) - \sin(1095)\cos(255)$

$\tan(\frac{3\pi}{2} + 15)\tan(-\pi + 15) - \sin(6\pi + 15)\cos(\frac{3\pi}{2} - 15)$

$-\cot 15 \times \tan 15 - \sin 15 \times (-\sin 15) = -1 + \sin^2 15$

$-\sin^2 15 - \cos^2 15 + \sin^2 15 = -\cos^2 15$

$k\cos^2 15 = -\cos^2 15 \to k = -1$

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$A = \sqrt{3}\ \mathrm{Cos}(210)\ \sin(243) - \sqrt{2}\ \sin(\underbrace{135}_{\frac{\sqrt{2}}{2}})\ \mathrm{Cos}(153)$

$\sqrt{3} \times -\frac{\sqrt{3}}{2} \times \sin\left(\frac{3\pi}{2} - 27\right) - \sqrt{2} \times \frac{\sqrt{2}}{2} \cos(\pi - 27)$

$-\frac{3}{2} \times (-\mathrm{Cos}\,27) - (-\mathrm{Cos}\,27) = \frac{3}{2} \mathrm{Cos}\,27 + \mathrm{Cos}\,27 = \frac{5}{2} \mathrm{Cos}\,27$

$\frac{\frac{5}{2}\mathrm{Cos}\,27}{\mathrm{Cos}\,27} = \frac{5}{2}$

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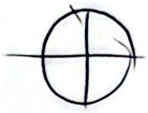

$f(x) = 16\ \mathrm{Cos}^2(3x)\ \mathrm{Cos}^2(6x)\ \mathrm{Cos}^2(12x)\ \mathrm{Cos}^2(24x)$

$f\left(\frac{\pi}{36}\right) = 16\ \mathrm{Cos}^2\left(\frac{\pi}{12}\right) \mathrm{Cos}^2\left(\frac{\pi}{6}\right) \mathrm{Cos}^2\left(\frac{\pi}{3}\right) \mathrm{Cos}^2\left(\frac{2}{3}\pi\right)$

$16\,\mathrm{Cos}^2\left(\frac{\pi}{12}\right)\left(\frac{\sqrt{3}}{2}\right)^2\left(\frac{1}{2}\right)^2\left(-\frac{1}{2}\right)^2 \rightarrow \cancel{16} \times \mathrm{Cos}^2\left(\frac{\pi}{12}\right) \times \frac{3}{4} \times \frac{1}{\cancel{4}} \times \frac{1}{\cancel{4}} = \frac{3}{4} \mathrm{Cos}^2\left(\frac{\pi}{12}\right)$

$\mathrm{Cos}^2 \frac{\pi}{12} = \frac{1 + \mathrm{Cos}\frac{\pi}{6}}{2} = \frac{1 + \frac{\sqrt{3}}{2}}{2} = \frac{2+\sqrt{3}}{4}$ $\qquad \frac{3}{4} \times \frac{2+\sqrt{3}}{4} = \boxed{\frac{6+3\sqrt{3}}{16}}$

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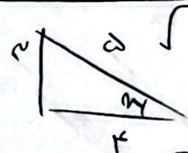
$\frac{1 - \sin x}{1 + \sin x} = 4 \rightarrow 4 + 4\sin x = 1 - \sin x$

$5 \sin x = -3 \rightarrow \sin x = -\frac{3}{5}$



$\sqrt{5^2 - 3^2} = 4$

$x \rightarrow$ ربع سوم $\rightarrow \mathrm{Cos}\,x < 0,\ \sin x < 0$

$\mathrm{Cos}\,x = -\frac{4}{5}$ ربع سوم

$\tan x = \frac{3}{4}$

$180 < x < 270$

$90 < \frac{x}{2} < 135$

$\tan \frac{x}{2} < 0$ (1)

$\tan x = \tan\left(\frac{x}{2} + \frac{x}{2}\right) = \frac{\tan\frac{x}{2} + \tan\frac{x}{2}}{1 - \tan\frac{x}{2} \times \tan\frac{x}{2}} = \frac{2\tan\frac{x}{2}}{1 - \tan^2\frac{x}{2}} = \frac{3}{4}$

$8\tan\frac{x}{2} = 3 - 3\tan^2\frac{x}{2} \rightarrow 3\tan^2\frac{x}{2} + 8\tan\frac{x}{2} - 3 = 0$

$3t^2 + 8t - 3 \xrightarrow{\text{روش}} t^2 + 8t - 9 \rightarrow (t+9)(t-1)$

$t_1 = -\frac{9}{3} = -3 = \tan\frac{x}{2} \rightarrow \checkmark$

$t_2 = \frac{1}{3} = \frac{1}{3} = \tan\frac{x}{2}$ غ ق ق (1)

$\boxed{\tan\frac{x}{2} = -3}$

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$\frac{\sin\theta}{1 - \mathrm{Cos}\,\theta} + \frac{1 + \mathrm{Cos}\,\theta}{\sin\theta}$

$1 + \mathrm{Cos}\,\theta = 2\mathrm{Cos}^2\frac{\theta}{2}$ $\qquad \sin\theta = 2\sin\frac{\theta}{2}\mathrm{Cos}\frac{\theta}{2}$

$1 - \mathrm{Cos}\,\theta = 2\sin^2\frac{\theta}{2}$

$\frac{\cancel{2\sin\frac{\theta}{2}}\ \mathrm{Cos}\frac{\theta}{2}}{\cancel{2} \times \cancel{\sin\frac{\theta}{2}} \times \sin\frac{\theta}{2}} + \frac{\cancel{2} \times \mathrm{Cos}\frac{\theta}{2}\ \cancel{\mathrm{Cos}\frac{\theta}{2}}}{\cancel{2}\sin\frac{\theta}{2} \times \cancel{\mathrm{Cos}\frac{\theta}{2}}} \Rightarrow 2\,\mathrm{Cot}\frac{\theta}{2} = k\,\mathrm{Cot}\frac{\theta}{2} \rightarrow \boxed{k = 2}$

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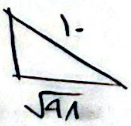
ربع دوم $\rightarrow \sin\alpha > 0$, $\mathrm{Cos}\,\alpha < 0$

$\mathrm{Cos}\left(\underbrace{\frac{11\pi}{4}}_{2\pi + \frac{3\pi}{4}} + \alpha\right) = \mathrm{Cos}\left(2\pi + \frac{3}{4}\pi + \alpha\right) = \mathrm{Cos}\left(\frac{3}{4}\pi + \alpha\right) = \underbrace{\mathrm{Cos}\frac{3}{4}\pi}_{-\frac{\sqrt{2}}{2}} \times \mathrm{Cos}\,\alpha - \underbrace{\sin\frac{3}{4}\pi}_{\frac{\sqrt{2}}{2}} \sin\alpha$

$\sin\alpha = \frac{\sqrt{2}}{10} \rightarrow \sqrt{2}$



$-\frac{\sqrt{2}}{2} \times -\frac{\sqrt{98}}{10} - \frac{\sqrt{2}}{\cancel{2}} \times \frac{\cancel{\sqrt{2}}}{10}$

$\mathrm{Cos}^2\alpha = \pm\frac{\sqrt{98}}{10} \xrightarrow{\text{ربع دوم}} -\frac{\sqrt{98}}{10}$

$\frac{\cancel{\sqrt{2}}}{\cancel{2}} \times \frac{7\sqrt{2}}{10} - \frac{1}{10} = \frac{6}{10}$