۱۳ و ۱۴

عکس کامروز

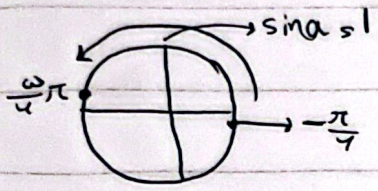
سوال ← $\cot a = \frac{\cos a}{\sqrt{1-\cos^2 a}} = \frac{\cos a}{\sqrt{(\sin a)^2}} = \frac{\cos a}{|\sin a|}$

$\frac{1}{|\cos a|} - \frac{|\sin a|}{\cos a} = \frac{1-\sin a}{|\cos a|} \rightarrow \frac{\cos a \pm \sin a |\cos a|}{\cos a |\cos a|} = \frac{1-\sin a}{|\cos a|}$

$\rightarrow \frac{|\cos a|(1-\sin a)}{\cos a} = 1-\sin a = 1 \rightarrow \cos a > 0, \sin a > 0$

← در ناحیه اول

سوال ۲ ←



$-\frac{\pi}{4} < x < \frac{5\pi}{4} \rightarrow -\frac{\pi}{2} < x < \frac{5\pi}{4}$

$\rightarrow -\frac{1}{2} < \sin x \leq 1 \rightarrow -\frac{1}{2} < \frac{m-1}{3} \leq 1$

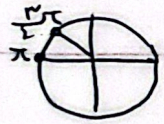
$\rightarrow -2 < m-1 \leq 4 \rightarrow -1 < m \leq 5$

$[5, -1)$

سوال ۳ ←

$\tan x + \cot x = \frac{1}{\sin \cos x} = -2 \rightarrow \sin \cos x = -\frac{1}{2}$

$2\pi < x < 3\pi \rightarrow \pi < x < \frac{3}{2}\pi$ ← در ناحیه دوم قرار دارد



$\rightarrow \frac{1}{\sin^2 x + \cos^2 x} = \frac{1}{(\sin x + \cos x)^2 - 2\sin x \cos x}$

$\rightarrow \frac{1}{(\sin x + \cos x)^2 - 2(-\frac{1}{2})} = \frac{1}{(\sin x + \cos x)^2 + 1} = \frac{1}{(\sin x + \cos x)^2 / (1 + (\sin x + \cos x)^2)}$

$\rightarrow (\sin x + \cos x)(1 + 1 - \frac{2}{3}) \rightarrow (\sin x + \cos x)(\frac{4}{3}) \rightarrow \frac{1}{(\sin x + \cos x)^2 (\frac{4}{3})}$

۴) $\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = -2 \rightarrow \sin x \cos x = -\frac{1}{2} = A$

$\frac{1}{\sin^2 x + \cos^2 x} = \frac{1}{(\sin x + \cos x)(1 - \sin x \cos x)}$

$A^2 = \sin^2 x + \cos^2 x + 2\sin x \cos x = \frac{1}{2}$

$\rightarrow A \begin{cases} \frac{1}{\sqrt{2}} \times \\ -\frac{1}{\sqrt{2}} \checkmark \end{cases}$

$\hookrightarrow \frac{-9}{4\sqrt{2}} = -\frac{9\sqrt{2}}{8}$

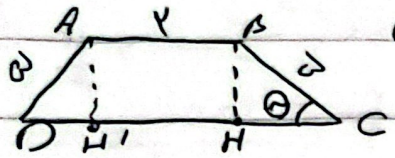
Scanned with CamScanner

$\cos\beta = \frac{3}{5} = \frac{6}{10}$

$\sin\beta = \frac{4}{5} = \frac{8}{10}$

$S = \frac{(2+8)}{2} \times 5 = 25$

سوال ۴ ←



$BH = \sin\theta \, BC \rightarrow 4 = \frac{4}{5} \times 5$

$BH = 4 \xrightarrow{\text{فیثاغورس}} HC = 3 \, , \, AH' = 3$

$\rightarrow DC = 10 \, , \, AB = 2 \, , \, BH = 4$

$\rightarrow S_{\text{ذوزنقه } ABCD} = \frac{(2+10) \times 4}{2} = 12$

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سوال ۵ ←

$\tan\left(\frac{3}{2}\pi + \alpha\right)\tan\alpha + \tan(\pi + \alpha) - \sin(2\pi + \alpha)\cos\left(\frac{3}{2}\pi - \alpha\right)$

$\frac{-\cot\alpha \times \tan\alpha}{-1} - \frac{(\sin\alpha \times -\sin\alpha)}{-\sin^2\alpha} \rightarrow -1 + \sin^2\alpha = -\cos^2\alpha$

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$\rightarrow k = -1$

سوال ۶ ←

$\sqrt{3}\cos(210)\sin\left(\frac{3}{2}\pi - 2\alpha\right) - \sqrt{2}\sin(135)\cos(\pi - 2\alpha)$

$\sqrt{3}\cos\left(-\frac{\sqrt{3}}{2}\right)\cos(-\cos 2\alpha) - \sqrt{2}\cos\frac{\sqrt{2}}{2}\cos(-\cos 2\alpha)$

$\frac{3}{2}\cos 2\alpha + \cos 2\alpha = \frac{5}{2}\cos 2\alpha \rightarrow$ (درست است)

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سوال ۷ ←

$14\cos^2\left(\frac{3}{24}\pi\right)\cos^2\left(\frac{4}{24}\pi\right)\cos^2\left(\frac{11}{24}\pi\right)\cos^2\left(\frac{12}{24}\pi\right)\cos^2\left(\frac{6\pi}{24}\right)\cos^2\left(\frac{\pi}{4}\right)\left(\frac{\pi}{3}\right)\left(\frac{\pi}{2}\right) \rightarrow \frac{1}{4} \times \frac{3}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{3}{64}$

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سوال ۸ ← $\cos\alpha = -\frac{3}{5}$ ، $\frac{\pi}{2} < \alpha < \pi$ ← $\frac{1-\sin x}{1+\sin x}$ را بیابید

$\sin x = 1 - \cos x \to \sin x = 1 - \left(-\frac{3}{5}\right) = \frac{8}{5}$

$\tan x = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}} \to \tan x = \frac{8}{6} \to \frac{8}{6} = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}$ (۶)

$4 - \frac{4}{3}\tan^2\frac{x}{2} = 8\tan\frac{x}{2} \to \frac{4}{3}\tan^2\frac{x}{2} + 8\tan\frac{x}{2} - 4$

$\to \tan^2\frac{x}{2} + 6\tan\frac{x}{2} - 3 \to \left(\tan\frac{x}{2} + 3\right)\left(\tan\frac{x}{2} - 1\right)$

$\to \tan\frac{x}{2} = -3$ ✓

$\to \tan\frac{x}{2} = \frac{1}{3}\alpha \to$ چون $\frac{x}{2}$ در ناحیه دوم است

$\to \tan\frac{x}{2} = -3$

سوال ۹ ← $\frac{\sin^2\theta + 1 - \cos^2\theta}{\sin\theta - \cos\theta\sin\theta} = \frac{2\sin^2\theta}{\sin\theta(1-\cos\theta)} = \frac{2\sin\theta}{1-\cos\theta}$ (۱)

$\tan^2\frac{\theta}{2} = \frac{1-\cos\theta}{2\sin\theta} = \frac{1-\cos\theta}{1+\cos\theta} \to k = 1$

سوال ۱۰ ← $\sin\alpha = \frac{\sqrt{2}}{10} \to \sin^2\alpha + \cos^2\alpha = 1 \to \left(\frac{\sqrt{2}}{10}\right)^2 + \cos^2\alpha = 1$ (۱۰)

$\to \cos^2\alpha = \frac{98}{100} \to \cos\alpha = -\frac{7\sqrt{2}}{10}$

$\cos\left(\frac{3}{4}\pi + \alpha\right) \to \cos\alpha\frac{3}{4}\pi + \cos\alpha = -\frac{\sqrt{2}}{2} + -\frac{7\sqrt{2}}{10}$

$\to \frac{-12\sqrt{2}}{10}$

$\cos\left(\frac{11\pi}{4} + \alpha\right) = -\left(\cos\alpha\cos\frac{\pi}{4} + \sin\alpha\sin\frac{\pi}{4}\right)$

$\to \frac{-\sqrt{2}}{2}(\cos\alpha + \sin\alpha)$ $\quad \cos\alpha = \frac{-7\sqrt{2}}{10}$

$\hookrightarrow -\frac{\sqrt{2}}{2}\left(\frac{-7\sqrt{2}}{10} + \frac{\sqrt{2}}{10}\right) = \frac{3}{5}$

Subó

۷) $f\left(\frac{\pi}{۲۴}\right) = ۱۴\cos^{۲}\left(\frac{\pi}{۱۲}\right)\cos^{۲}\left(\frac{\pi}{۶}\right)\cos^{۲}\left(\frac{\pi}{۴}\right)\cos^{۲}\left(\frac{۲\pi}{۴}\right)$

$\cos^{۲}\frac{\pi}{۱۲} = \frac{۱+\cos\frac{\pi}{۶}}{۲} = \frac{۲+\sqrt{۳}}{۴}$

$۱۴\left(\frac{۲+\sqrt{۳}}{۴}\right)\times\frac{۳}{۴}\times\frac{۱}{۴}\times\frac{۱}{۴}$

$= \frac{۳(۲+\sqrt{۳})}{۱۶}$

۹) $\frac{\sin^{۲}\theta+(۱-\cos^{۲}\theta)}{(۱-\cos\theta)\sin\theta} = \frac{۲\sin^{۲}\theta}{\sin\theta(۱-\cos\theta)} = \frac{۲\times۲\times\sin\frac{\theta}{۲}\cos\frac{\theta}{۲}}{۲\sin^{۲}\frac{\theta}{۲}} = ۲\cot\frac{\theta}{۲}$

$\rightarrow k = ۲$