حل کامل

سوال ←

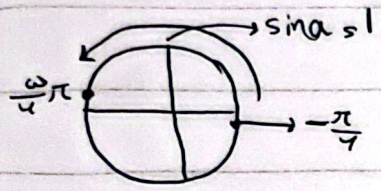
$$\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} = \frac{\cos\alpha}{\sqrt{(\sin\alpha)^2}} = \frac{\cos\alpha}{|\sin\alpha|}$$

$$\frac{1}{|\cos\alpha|} - \frac{|\sin\alpha|}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \rightarrow \frac{\cos\alpha \pm \sin\alpha|\cos\alpha|}{\cos\alpha|\cos\alpha|} = \frac{1-\sin\alpha}{|\cos\alpha|}$$

$$\rightarrow \frac{|\cos\alpha|(1-\sin\alpha)}{\cos\alpha} = 1-\sin\alpha = 1 \rightarrow \cos\alpha > 0 \text{ , } \sin\alpha > 0$$

← در ناحیه اول

سوال ۲ ←



$$-\frac{\pi}{4} < x < \frac{3\pi}{4} \rightarrow -\frac{\pi}{4} < x < \frac{5\pi}{4}$$

$$\rightarrow -\frac{1}{2} < \sin x < 1 \rightarrow -\frac{1}{2} < \frac{1-m}{3} < 1$$

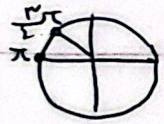
$$\rightarrow -2 < m-1 < 3 \rightarrow -1 < m < 4$$

$$[-1, 4)$$

سوال ۳ ←

$$\tan x + \cot x = \frac{1}{\sin\cos x} = -2 \rightarrow \sin\cos x = -\frac{1}{2}$$

$$\pi < x < \frac{3}{2}\pi \rightarrow \text{در ناحیه دوم قرار دارد} \rightarrow \frac{3}{4}\pi < x < \frac{3}{2}\pi$$

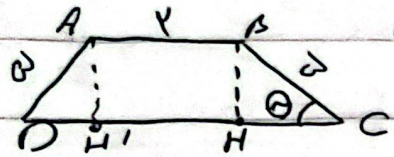


$$\rightarrow \frac{1}{\sin^2 x + \cos^2 x} = \frac{1}{(\sin x + \cos x)^2 - 2\sin x \cos x}$$

$$\rightarrow \frac{1}{(\sin x + \cos x)^2 - 2\left(-\frac{1}{2}\right)} = \frac{1}{(\sin x + \cos x)^2 + 1} = \frac{1}{(\sin x + \cos x)^2 + 1}$$

$$\rightarrow (\sin x + \cos x)\left(1 + 1 - \frac{2}{3}\right) \rightarrow (\sin x + \cos x)\left(\frac{4}{3}\right) \rightarrow \frac{1}{\left(\frac{4}{3}\right)(\sin x + \cos x)}$$

سوال ۵ ←



$BH = \sin\theta BC \rightarrow \frac{\sqrt{3}}{2}\alpha 5 = 4$

$BH = 4 \xrightarrow{\text{فیثاغورس}} HC = 8, AH' = 8$

$\rightarrow DC = 10, AB = 2, BH = 4$

$\rightarrow S_{\text{ذوزنقه}} ABCD = \frac{(2+10)\times 4}{2} = 24$

سوال ۶ ←

$\tan(\frac{3}{2}\pi+\theta)\alpha\tan(\pi+\theta) - \sin(2\pi+\theta)\cos(\frac{3}{2}\pi-\theta)$

$-\cot\theta \times \tan\theta - (\sin\theta\times\sin\theta) \rightarrow -1+\sin^2\theta = -\cos^2\theta$

$\rightarrow k = -1$

سوال ۶ ←

$\sqrt{3}\cos(210)\sin(\frac{3}{2}\pi-2\theta) - \sqrt{2}\sin(135)\cos(\pi-2\theta)$

$\sqrt{3}\alpha(-\frac{\sqrt{3}}{2})\alpha(-\cos 2\theta) - \sqrt{2}\alpha\frac{\sqrt{2}}{2}\alpha(-\cos 2\theta)$

$\frac{3}{2}\cos 2\theta + \cos 2\theta = \frac{5}{2}\cos 2\theta \rightarrow$ درست

سوال ۷ ←

$16\cos^2(\frac{3}{24}\pi)\cos^2(\frac{7}{24}\pi)\cos^2(\frac{11}{24}\pi)\cos^2(\frac{15}{24}\pi)\cos^2(\frac{19}{24}\pi)\cos^2(\frac{23}{24}\pi)$

$(\frac{\pi}{12})(\frac{\pi}{4})(\frac{\pi}{3})(\frac{\pi}{2}) \rightarrow \frac{1}{4}\times\frac{3}{4}\times\frac{1}{4} = \frac{3}{64}$

سوال ۸ ←

$$\frac{1-\sin x}{1+\sin x} \to \frac{1-\sin x}{1+\sin x} = \frac{1}{4} \to 4-4\sin x = 1+\sin x \to \sin x = \frac{3}{5} \to \cos x = -\frac{4}{5}$$

$$\tan x = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}} \to \tan x = \frac{3}{4} \to \frac{3}{4} = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}$$

$$3 - \frac{3}{2}\tan^2\frac{x}{2} = 4\tan\frac{x}{2} \to \frac{3}{2}\tan^2\frac{x}{2} + 4\tan\frac{x}{2} - 3$$

$$\to \tan^2\frac{x}{2} + 8\tan\frac{x}{2} - 9 \to \left(\tan\frac{x}{2}+9\right)\left(\tan\frac{x}{2}-1\right)$$

$\to \tan\frac{x}{2} = -3$ ✓

$\to \tan\frac{x}{2} = \frac{1}{3}$ ✗ چون $\frac{x}{2}$ در ناحیه دوم است

$\to \tan\frac{x}{2} = -3$

سوال ۹ ←

$$\frac{\sin^2\theta + 1 - \cos^2\theta}{\sin\theta - \cos\theta\sin\theta} = \frac{2\sin^2\theta}{\sin\theta(1-\cos\theta)} = \frac{2\sin\theta}{1-\cos\theta}$$

$$\tan^2\frac{\theta}{2} = \frac{1-\cos\theta}{2\sin\theta} = \frac{1-\cos\theta}{1+\cos\theta} \to k = 1$$

سوال ۱۰ ←

$$\sin\alpha = \frac{\sqrt{2}}{10} \to \sin^2\alpha + \cos^2\alpha = 1 \to \left(\frac{\sqrt{2}}{10}\right)^2 + \cos^2\alpha = 1$$

$$\to \cos^2\alpha = \frac{98}{100} \to \cos\alpha = -\frac{7\sqrt{2}}{10}$$

$$\cos\left(\frac{3}{4}\pi + \alpha\right) \to \cos\alpha\frac{3}{4}\pi + \cos\alpha = -\frac{\sqrt{2}}{2} + -\frac{7\sqrt{2}}{10}$$

$$\to \frac{-12\sqrt{2}}{10}$$