۱) دبیا سوالات تجربی — تشریحی — ۱۴ مرداد

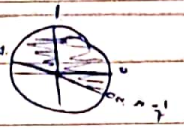
$\frac{1}{\sqrt{\cos^2\alpha}} = \frac{1}{\cot\alpha} \times \frac{1-\sin\alpha}{|\cos\alpha|} \times \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \times \frac{4\sin\alpha}{\cos\alpha} = \frac{4-4\sin\alpha}{|\cos\alpha|}$

$\cos\alpha \geq 0$ ناصحیح

$\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} = \frac{\cos\alpha}{|\sin\alpha|} \rightarrow \frac{\cos\alpha}{|\sin\alpha|} = \frac{\cos\alpha}{\sin\alpha} \rightarrow \sin\alpha > 0$

۲)

$-\frac{\pi}{12} < 2x < \frac{5\pi}{12} \rightarrow -\frac{\pi}{6} < x < \frac{5\pi}{6}$



$-\frac{1}{2} < \sin 2x \leq 1 \rightarrow -\frac{1}{2} < \frac{m-1}{4} \leq 1 \rightarrow -2 < m-1 \leq 4 \rightarrow -1 < m \leq 5$

۳)

$\tan x + \cot x = -4 \rightarrow \frac{\cos^2 x + \sin^2 x}{\cos x \sin x} = -4 \rightarrow \cos x \sin x = -\frac{1}{4}$

$\frac{3\pi}{4} < x < \frac{5\pi}{4} \rightarrow \frac{3\pi}{4} < x < \pi$ ($\sin x > 0$, $\cos x < 0$)

$(\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2\cos x \sin x = \frac{1}{4} \rightarrow \sin x + \cos x = -\frac{\sqrt{2}}{2}$

$(\sin x - \cos x)^2 = \sin^2 x + \cos^2 x - 2\sin x \cos x = \frac{5}{4} \rightarrow \sin x - \cos x = \frac{\sqrt{10}}{2}$

$\sin x = \frac{\sqrt{10}-\sqrt{2}}{4}$, $\cos x = \frac{-\sqrt{10}-\sqrt{2}}{4}$

$\frac{1}{\sin^4 x + \cos^4 x} = \frac{1}{(\sin^2 x + \cos^2 x)^2 - 2\cos^2 x \sin^2 x} = \frac{1}{(\sin x + \cos x)^4 - 2\sin^2 x(\sin x + \cos x)^2} = \frac{1}{\left(\frac{-\sqrt{2}}{2}\right)^4 - 2\left(\frac{-1}{2}\right)\left(\frac{-\sqrt{10}}{2}\right)} = \frac{1}{\frac{4}{16} - \frac{2\sqrt{10}}{16}} = \frac{8}{4-2\sqrt{10}}$

$= \frac{8(4+2\sqrt{10})}{16-40} = \frac{4+2\sqrt{10}}{-12} = \frac{-2-\sqrt{10}}{6}$

۴)

$\cos\theta = \frac{4}{5} \rightarrow \frac{4}{5} = \frac{DH}{AD} \rightarrow DH = 3$ ، $\hat{ADH} \cong \hat{BH'C}$ (ض ز ض)

$BH' = 3$

$\hat{ADH}$: $AH^2 + DH^2 = AD^2 \rightarrow AH = 4$ ، $H'C = 4$ ، $ABH'H$ مستطیل ، $AB = HH' = 2$

$DC = 3 + 2 + 3 = 8$

$\cos\omega = \frac{2^2 + 8^2 - (2\sqrt{5})^2}{2 \times 2 \times 8} = \frac{4+64-20}{32} = \frac{48}{32} = \frac{3}{2}$

$\tan(\pi\omega)\tan(-4\omega) - \sin(-2\omega)\cos(2\omega) = \tan\left(\frac{\pi x}{2} + \omega\right)\tan(-x+\omega) - \sin(4x+\omega)\cos\left(\frac{\pi x}{2} - \omega\right)$

$= (-\cot\omega)(\tan\omega) - \sin\omega\cos\omega = \sin^2\omega - 1 = -\cos^2\omega \rightarrow k = -1$

۵)

$\sqrt{3}\cos 150^\circ \sin 240^\circ - \sqrt{2}\sin 120^\circ \cos 150^\circ = \sqrt{3}\cdot\left(-\frac{\sqrt{3}}{2}\right)\cdot\sin\left(\frac{3\pi}{2} - 27\right) - \sqrt{2}\cdot\frac{\sqrt{2}}{2}\cdot\cos(\pi - 27)$

$= -\frac{1}{2}(-\cos 27) - \left(-\cos 27\right) = \frac{\cos 27}{2} + \cos 27 = \frac{3\cos 27}{2} \rightarrow$ برابر $\frac{3}{2}$

۶) $A = \sqrt{3}\sqrt{7}\cdot\frac{-\sqrt{3}}{2}\sqrt{7}\sin(270-27) - \sqrt{2}\sqrt{7}\frac{\sqrt{2}}{2}\cos(180-27)$

$\rightarrow \frac{3}{2}\cos(27)$ $\rightarrow$ برابر $\frac{3}{2}$

$$14\cos^2\left(\frac{\pi}{12}\right)\cos^2\left(\frac{\pi}{6}\right)\cos^2\left(\frac{5\pi}{12}\right)\cos^2\left(\frac{\pi}{3}\right) = 14\sin^2\left(\frac{\pi}{12}\right)\cos^2\left(\frac{\pi}{6}\right)\cos^2\left(\frac{\pi}{3}\right)\cos^2\left(\frac{5\pi}{12}\right) \quad (\text{V}$$

$$= \frac{14\sin^2\left(\frac{\pi}{12}\right)\cos^2\left(\frac{\pi}{12}\right)\cos^2\left(\frac{\pi}{3}\right)\cos^2\left(\frac{5\pi}{12}\right)}{\sin^2\left(\frac{\pi}{6}\right)} = \frac{4\sin^2\left(\frac{\pi}{12}\right)\cos^2\left(\frac{\pi}{12}\right)\cos^2\left(\frac{\pi}{3}\right)}{\sin^2\left(\frac{\pi}{6}\right)} = \frac{4\sin^2\left(\frac{\pi}{6}\right)\cos^2\left(\frac{\pi}{6}\right)}{\sin^2\left(\frac{\pi}{6}\right)}$$

$$= \frac{4\sin^2\left(\frac{\pi}{6}\right)}{\sin^2\left(\frac{\pi}{4}\right)} = \frac{4\sin^2\left(\frac{\pi}{6}\right)}{\sin^2\left(\frac{\pi}{4}\right)} = \frac{4\left(\frac{1}{2}\right)^2}{\left(\frac{\sqrt{2}}{2}\right)^2} = \frac{4\left(\frac{1}{4}\right)}{\frac{2}{4}} = \frac{1}{\frac{1}{2}} = 2 \quad (1)$$

$$\frac{1-\sin x}{1+\sin x} = 2 \rightarrow 1-\sin x = 2+2\sin x \Rightarrow 3\sin x = -1 \rightarrow \sin x = -\frac{1}{3} \quad (\Lambda$$

$$\tan^2\left(\frac{\alpha}{2}\right) = \frac{1-\cos\alpha}{1+\cos\alpha} = \frac{1+\frac{1}{3}}{1-\frac{1}{3}} = \frac{\frac{4}{3}}{\frac{2}{3}} = 2 \rightarrow \tan\left(\frac{\alpha}{2}\right) = \sqrt{2}$$

$\cos\alpha = -\frac{2}{3}$ (18)

$$\frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = \frac{1}{\tan\left(\frac{\theta}{2}\right)} + \frac{1}{\tan\left(\frac{\theta}{2}\right)} = 2\cot\left(\frac{\theta}{2}\right) \rightarrow k = 2 \quad (9$$

(8)

$$\sin\alpha = \frac{\sqrt{2}}{10} \Rightarrow \sin^2\alpha + \cos^2\alpha = 1 \Rightarrow \frac{2}{100} + \cos^2\alpha = 1 \Rightarrow \cos^2\alpha = \frac{98}{100} \Rightarrow \cos\alpha = -\frac{\sqrt{98}}{10} \quad (10$$

(9)

$$\cos\left(\frac{11\pi}{12}+\alpha\right) = \cos\left(2\pi + \frac{3\pi}{12} + \alpha\right) = \cos\left(\frac{3\pi}{12}+\alpha\right) = \cos\left(\frac{\pi}{4}+\alpha\right) = \cos\alpha\cos\frac{\pi}{4} - \sin\alpha\sin\frac{\pi}{4}$$

$$= \left(-\frac{\sqrt{98}}{10}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{2}}{10}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{-7\sqrt{2}-\sqrt{2}}{20} = \frac{-8\sqrt{2}}{20} = -\frac{4}{5}$$

(V) $\tan\left(\frac{\pi}{24}\right) = 14\cos^2\left(\frac{\pi}{12}\right)\cos^2\left(\frac{\pi}{6}\right)\cos^2\left(\frac{\pi}{3}\right)\cos^2\left(\frac{5\pi}{12}\right)$

$\cos^2\frac{\pi}{12} = \frac{1+\cos\frac{\pi}{6}}{2} = \frac{2+\sqrt{3}}{4}$

$14\left(\frac{2+\sqrt{3}}{4}\right)\times\frac{3}{4}\times\frac{1}{4}\times\frac{1}{4} = \frac{3(2+\sqrt{3})}{14}$

(Λ) $1-\sin\alpha = 2+2\sin\alpha \rightarrow \sin\alpha = -\frac{1}{3}$, $\cos\alpha = -\frac{2}{3}$ $\quad \cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2} = \frac{1}{10}$

$1+\tan^2\frac{\alpha}{2} = \frac{1}{\cos^2\frac{\alpha}{2}} \rightarrow \tan\frac{\alpha}{2} = \pm 3$ $\quad \frac{\pi}{2} < \frac{\alpha}{2} < \pi$ $\quad \tan\frac{\alpha}{2} = -3$