باران کاظمی — یازدهم C — تکلیف شماره ۲۸

۱)

$$\frac{1}{\sqrt{\cos^2\alpha}}-\frac{1}{\cot\alpha}=\frac{1-\sin\alpha}{|\cos\alpha|}\Rightarrow\frac{1}{|\cos\alpha|}-\frac{\sin\alpha}{\cos\alpha}=\frac{1-\sin\alpha}{|\cos\alpha|}$$

$$\frac{-1-\sin\alpha}{\cos\alpha}=\left\langle\begin{array}{l}\cancel{\frac{1-\sin\alpha}{\cos\alpha}}\\ \frac{-1+\sin\alpha}{\cos\alpha}\end{array}\right.\quad \text{نیست}$$

$$\to\frac{1-\sin\alpha}{\cos\alpha}=\frac{1-\sin\alpha}{|\cos\alpha|}\to\boxed{\cos\alpha>0}\ ①$$

$$\cot\alpha=\frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}}\to\frac{\cos\alpha}{\sin\alpha}=\frac{\cos\alpha}{|\sin\alpha|}\to\boxed{\sin\alpha>0}\ ②$$

$\xrightarrow{①,②}$ ناحیه اول

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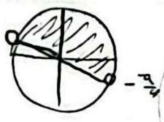
۲)

$$-\frac{\pi}{12}<x<\frac{5\pi}{12}\to-\frac{\pi}{6}<2x<\frac{5\pi}{6}$$

$$-\frac{1}{2}<\sin 2x\le 1\to-\frac{1}{2}<\frac{m-1}{4}\le 1$$

$$-2<m-1\le 4\to\boxed{-1<m\le 5}$$



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۳)

$$\frac{1}{\sin^3x+\cos^3x}=\frac{1}{\underbrace{(\sin x+\cos x)}_{\sqrt{\frac{1}{3}}}\underbrace{(\cancel{\sin^2x+\cos^2x}-\sin x\cos x)}_{1+\frac{1}{3}=\frac{4}{3}}}=\frac{1}{\sqrt{\frac{1}{3}}\left(\frac{4}{3}\right)}=\frac{1}{\frac{\sqrt{3}}{3}\times\frac{4}{3}}=\frac{9}{4\sqrt{3}}=\frac{9\sqrt{3}}{3\times4}=\boxed{\frac{3\sqrt{3}}{4}}$$

$$\tan x+\cot x=-3\to\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}=-3\to\frac{\cancel{\sin^2x+\cos^2x}}{\sin x\cos x}=-3\to\sin x\cos x=-\frac{1}{3}$$

$$(\sin x+\cos x)^2=\underbrace{\cancel{\sin^2x+\cos^2x}}_{1}+\underbrace{2\sin x\cos x}_{-\frac{2}{3}}=\frac{1}{3}\to\sin x+\cos x=\sqrt{\frac{1}{3}}$$

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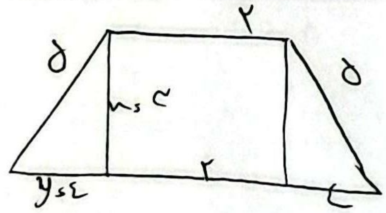
۴)



$$\frac{h}{5}=0{,}6\to h=3$$

$$\sin\theta=1-0{,}36=0{,}64\to\sin\theta=0{,}8$$

$$\frac{y}{5}=0{,}8\to y=4$$

$$S=\frac{3\times12}{2}=\boxed{18}$$

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۵)

$$\tan(285^\circ)\tan(-165)-\sin(1095)\cos(255)$$

$$-\tan(285)\tan(165)-\sin 1095\cos(255)$$

$$-\tan\left(\frac{3\pi}{2}+15\right)\tan(\pi-15)-\sin(6\pi+15)\cos\left(\frac{3\pi}{2}-15\right)$$

$$-\underbrace{\cancel{\cot 15}\,\cancel{\tan 15}}_{1}+\sin15\sin15=\sin^2 15-1=\left.\begin{array}{l}-\cos^2 15\\=k\cos^2 15\end{array}\right\}\to\boxed{k=-1}$$

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۶)

$$\sqrt{3}\cos(210)\sin(243)-\sqrt{2}\sin(135)\cos(153)$$

$$=\sqrt{3}\cos(210)\sin\left(\frac{3\pi}{2}-27\right)-\sqrt{2}(\sin(135))(\cos(\pi-27))$$

$$=\sqrt{3}\left(-\frac{\sqrt{3}}{2}\right)(-\cos27)-(\sqrt{2})\left(\frac{\sqrt{2}}{2}\right)(-\cos27)$$

$$=\frac{3}{2}\cos27+\cos27=\frac{5}{2}\cos27\longrightarrow\frac{\frac{5}{2}\cos27}{\cos27}=\boxed{\frac{5}{2}}$$

(۷

$$f(x) = 16\left[\frac{\sin^2(3x)}{\sin^2(3x)}\cos^2(3x)\cos^2(6x)\cos^2(12x)\cos^2(24x)\right]$$

$4\sin^2(6x)$, $4\sin^2(12x)$, $4\sin^2(24x)$, $4\sin^2(48x)$

$$\frac{\cancel{16}\sin^2\left(\frac{48\pi}{72}\right)}{\cancel{4\times4}\times4\times4\sin^2\left(\frac{3\pi}{72}\right)} = \frac{\frac{3}{4}}{4\times4\times\frac{2-\sqrt3}{4}} = \frac{3}{16(2-\sqrt3)}\times\frac{(2+\sqrt3)}{(2+\sqrt3)} = \boxed{\frac{6+3\sqrt3}{16}}$$

$$\sin^2\left(\frac{\pi}{12}\right) = \frac{1-\cos\left(\frac{\pi}{6}\right)}{2} = \frac{1-\frac{\sqrt3}{2}}{2} = \frac{2-\sqrt3}{4}$$

(۸

$$\frac{1-\sin x}{1+\sin x} = 4 \to 1-\sin x = 4+4\sin x \to 5\sin x = -3 \to \sin x = \frac{-3}{5}$$

ربع ۳

$\cos x = \frac{4}{5}$ غ ق ق $\qquad \boxed{\frac{-4}{5}}$

$$\tan x = \frac{-\frac{3}{5}}{-\frac{4}{5}} = \frac{3}{4}$$

$$\tan x = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}} \to \frac{3}{4} = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}} \to 3-3\tan^2\frac{x}{2} = 8\tan\frac{x}{2}$$

$$3\tan^2\frac{x}{2} + 8\tan\frac{x}{2} - 3 = 0 \to \frac{-8\pm\sqrt{64+36}}{6}$$

$\sqrt{64+36} = 10$

-3 غ ق ق

$\frac{1}{3} \to \boxed{\tan\frac{x}{2} = \frac{1}{3}}$

(۹

$$\frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} \to \frac{\sin^2\theta + 1-\cos^2\theta}{\sin\theta(1-\cos\theta)} = \frac{(\sin^2\theta + \sin^2\theta + \cancel{\cos^2\theta} - \cancel{\cos^2\theta})}{\sin\theta(1-\cos\theta)}$$

$$\Rightarrow \frac{2\sin^2\theta}{\cancel{\sin\theta}(1-\cos\theta)} \to \frac{2\sin\theta}{1-\cos\theta} = \frac{4\cancel{\sin\frac{\theta}{2}}\cos\frac{\theta}{2}}{\sin^2\frac{\theta}{2}} = 4\cot\left(\frac{\theta}{2}\right)$$

$\boxed{k=4}$

(۱۰

$$\cos\left(\frac{11\pi}{4}+\alpha\right) = \cos\frac{11\pi}{4}\cos\alpha - \sin\frac{11\pi}{4}\sin\alpha$$

ربع ۲

$$= \left(-\frac{\sqrt2}{2}\right)\left(-\frac{7\sqrt2}{10}\right) - \left(\frac{\sqrt2}{2}\right)\left(\frac{\sqrt2}{10}\right) = \frac{7}{10} - \frac{1}{10} = \frac{6}{10} = \boxed{\frac{3}{5}}$$

$$\cos\frac{11\pi}{4} = \underbrace{\cos\left(\pi + \frac{7\pi}{4}\right)}_{\cos\pi\cos\frac{7\pi}{4} - \underbrace{\sin\pi\sin\frac{7\pi}{4}}_{0}} = -\frac{\sqrt2}{2}$$

$-1\left(\frac{\sqrt2}{2}\right)$

$$\cos\alpha = -\frac{7\sqrt2}{10}$$