سنجش حسابان   یازدهم پنجشنبه دفتر ۲

(۱)

$$\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} \qquad \frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|}$$

$1-\cos^2\alpha$ → $\sin^2\alpha$

$$\frac{1}{|\cos\alpha|} - \frac{\cos\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|}$$

$$\cot\alpha = \frac{\cos\alpha}{|\sin\alpha|} \rightarrow \sin\alpha \geq 0 \quad I$$

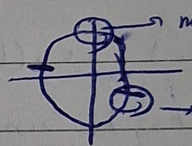
$II \quad \cos\alpha \geq 0$

$I \cap II =$ ربع اول

(۲)

$$\sin 2\alpha = \frac{m-1}{4} \qquad \frac{-\pi}{12} < \alpha < \frac{5\pi}{12} \qquad \frac{-2\pi}{12} < 2\alpha < \frac{10\pi}{12}$$

$$\rightarrow -\frac{\pi}{6} < 2\alpha < \frac{5\pi}{6} \qquad -\frac{1}{2} < \sin 2\alpha \leq 1$$



$$-\frac{1}{2} < \frac{m-1}{4} \leq 1$$

$$-1 < m \leq 5 \rightarrow (-1, 5]$$

(۳)

$$\tan\alpha + \cot\alpha = -3 \qquad 3\pi < 4\alpha < 4\pi \qquad \frac{1}{\sin^3\alpha + \cos^3\alpha}$$

$$\frac{\frac{1}{\sin^2\alpha+\cos^2\alpha}}{\sin\alpha\cos\alpha} = -3 \rightarrow \sin\alpha\cos\alpha = \frac{-1}{3}$$

$$\frac{3\pi}{4} < \alpha < \pi \rightarrow 0 < \sin\alpha < \frac{\sqrt{2}}{2}, \quad -1 < \cos\alpha < -\frac{\sqrt{2}}{2}$$

$$\sin^3\alpha + \cos^3\alpha = (\sin\alpha + \cos\alpha)(\sin^2\alpha + \cos^2\alpha - \sin\alpha\cos\alpha) = \left(-\frac{\sqrt{3}}{3}\right)\left(\frac{4}{3}\right)$$

$$= -\frac{4\sqrt{3}}{9} \qquad A = \sin\alpha + \cos\alpha < 0 \rightarrow (\sin\alpha+\cos\alpha)^2 = \underbrace{\sin^2\alpha+\cos^2\alpha}_{1} + 2\sin\alpha\cos\alpha$$

(مربع)

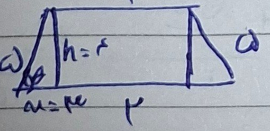
$$A^2 = 1 - \frac{2}{3} = \frac{1}{3} \rightarrow A = \pm\sqrt{\frac{1}{3}}$$

$$\rightarrow \cos\alpha + \sin\alpha < 0 \qquad \frac{1}{\sin^3\alpha + \cos^3\alpha} = \frac{1}{\frac{-4}{3\sqrt{3}}} = \frac{-3\sqrt{3}}{4}$$

(معکوس)

(۴)

$$\cos\theta = 0.6$$



$$\frac{6}{10} = \frac{x}{5}$$

$$x = 3 \Rightarrow h = 4$$

$$\frac{\left(2 + (2 + 2\times 3)\right) \times 4}{2} = 20$$

$$S_{\text{ذوزنقه}} = \frac{\text{(ارتفاع)} \times \text{مجموع قاعده ها}}{2}$$

(5)

$k \cos^2 15 = \tan(285^\circ)\tan(-165^\circ) - \sin(1095^\circ)\cos(255^\circ)$

$\cos(255^\circ) = \cos(270 - 15) = -\sin 15$

$\tan(285^\circ) = \tan(270 + 15) = -\cot 15^\circ$

$\tan(-165^\circ) = -\tan(180 - 15) = \tan 15^\circ$

$-\sin(1095^\circ) = -\sin(\underbrace{1080}_{6\pi} + 15) = -\sin 15$ ✓

$\underbrace{(-\cot 15)(\tan 15)}_{-1} + (\sin^2 15) = -1 + \sin^2 15^\circ$

$-(1 - \sin^2 15) = -\cos^2 15^\circ$

$\boxed{k = -1}$

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(6)

$A = \sqrt{3}\cos(210^\circ)\sin(243^\circ) - \sqrt{2}\sin(135^\circ)\cos(153^\circ) = k\cos 27^\circ$

$\cos(210^\circ) = \cos(180^\circ + 30^\circ) = -\cos 30^\circ = \frac{-\sqrt{3}}{2}$

$\sin(243^\circ) = \sin(270^\circ - 27^\circ) = -\cos 27^\circ$

$\sin(135^\circ) = \sin(180^\circ - 45^\circ) = \sin 45 = \frac{\sqrt{2}}{2}$

$\cos(153^\circ) = \cos(180^\circ - 27^\circ) = -\cos 27$

$\longrightarrow (\sqrt{3})\left(-\frac{\sqrt{3}}{2}\right)(-\cos 27^\circ) - \sqrt{2}\left(\frac{\sqrt{2}}{2}\right)(-\cos 27^\circ)$

$= \frac{3}{2}\cos 27^\circ + \cos 27^\circ = \frac{5}{2}\cos 27^\circ$ $\quad \boxed{k = \frac{5}{2}}$

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(7)

$f\left(\frac{\pi}{36}\right) = ?$ $\quad f(a) = 16\cos^2(3a)\cos^2(9a)\cos^2(12a)\cos^2(24a)$

$3a \to \frac{\pi}{12}$, $9a \to \frac{\pi}{4}$, $12a \to \frac{\pi}{3}$, $24a \to \frac{2\pi}{3}$

$16 \times \frac{2+\sqrt{3}}{4} \times \frac{1}{2} \times \frac{3}{4} \times \frac{1}{4} = \frac{6 + 3\sqrt{3}}{16}$

$\cos^2\left(\frac{\pi}{12}\right) = \frac{1 + \cos\frac{\pi}{6}}{2} = \frac{1 + \frac{\sqrt{3}}{2}}{2} = \frac{2 + \sqrt{3}}{4}$

(۸)

$\frac{1-\sin\alpha}{1+\sin\alpha} = ۴ \longrightarrow$ (۹ن ا) $\tan\frac{\alpha}{۲}$

$1-\sin\alpha = ۴ + ۴\sin\alpha \rightarrow ۵\sin\alpha = -۳$

$\sin\alpha = \frac{-۳}{۵}$

$\sin^۲\alpha + \cos^۲\alpha = 1$

$\frac{۹}{۲۵} + \cos^۲\alpha = 1 \rightarrow \cos\alpha = -\frac{۴}{۵}$ غق

$\cos\alpha = \frac{۴}{۵}$ ققغ

$\tan\frac{\alpha}{۲} = \frac{\sin\alpha}{1+\cos\alpha} = \frac{-\frac{۳}{۵}}{1+\frac{-۴}{۵}} = \frac{-\frac{۳}{۵}}{\frac{1}{۵}} = \boxed{-۳}$

(۹)

$\frac{\sin\alpha}{1-\cos\alpha} + \frac{1+\cos\alpha}{\sin\alpha}$ $\underset{?}{k}\cot\frac{\alpha}{۲}$

$\frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha} = \tan\frac{\alpha}{۲}$

$\cot\frac{\alpha}{۲} + \cot\frac{\alpha}{۲} = ۲\cot\frac{\alpha}{۲} = k\cot\frac{\alpha}{۲} \rightarrow \boxed{k=۲}$

(۱۰)

$\sin = \frac{\sqrt{۲}}{۱۰}$ $\cos = -\frac{\sqrt{۹۸}}{۱۰}$

$\cos\left(\frac{۱۱\pi}{۴}+\alpha\right) = \cos\left(۳\pi - \frac{\pi}{۴} + \alpha\right) = \cos\left(۳\pi - \left(\frac{\pi}{۴} - \alpha\right)\right)$

$-\cos\left(\frac{\pi}{۴} - \alpha\right) = -\left(\cos\frac{\pi}{۴}\cos\alpha + \sin\frac{\pi}{۴}\sin\alpha\right)$

$-\left(\frac{\sqrt{۲}}{۲} \times \frac{-\sqrt{۹۸}}{۱۰} + \frac{\sqrt{۲}}{۱۰} \times \frac{\sqrt{۲}}{۱۰}\right) = -\left(-\frac{\sqrt{۱۹۶}}{۲۰} + \frac{۲}{۲۰}\right)$

$= -\left(\frac{-۱۴+۲}{۲۰}\right) = -\left(\frac{-۱۲}{۲۰}\right) = \boxed{+\frac{۳}{۵}}$