روژانو صدرالعلمایی — یازدهم دختر C — تکلیف ۲۸

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چون صورت و مخرج ⊕ هستند => $|\sin\alpha|$ باید مثبت باشد => $\sin\alpha > 0$

$$\cot\alpha = \frac{\cos\alpha}{\sqrt{\underbrace{1-\cos^2\alpha}_{\sin^2\alpha}}} \quad , \quad \frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|}$$

=> ربع اول

$$\cot\alpha = \frac{\cos\alpha}{|\sin\alpha|} \qquad \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{\cos\alpha} \Rightarrow \cos\alpha > 0$$

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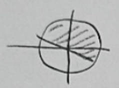
$$\sin 2x = \frac{m-1}{4} \qquad -\frac{\pi}{6} < 2x < \frac{5\pi}{6} \Rightarrow -\frac{1}{2} < \underbrace{\sin 2x}_{\frac{m-1}{4}} \leq 1$$

$$-\frac{\pi}{12} < x < \frac{5\pi}{12}$$



$$-2 < m-1 \leq 4 \rightarrow \boxed{-1 < m \leq 5}$$

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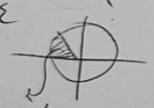
$$\tan x + \cot x = -3$$

$$3\pi < 4x < 4\pi$$

$$\frac{3\pi}{4} < x < \pi$$



اندازه $\cos\alpha$ بزرگتر از $\sin\alpha$

$$\frac{1}{\sin^3 x + \cos^3 x} = \frac{1}{\underbrace{(\sin x+\cos x)}_{-\frac{\sqrt{3}}{3}}\underbrace{(\sin^2+\cos^2 - \sin x\cos x)}_{1+\frac{1}{3}=\frac{4}{3}}} = \boxed{\frac{9}{-4\sqrt{3}}}$$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{1}{\sin x\cos x} = -3 \qquad \sin x\cos x = -\frac{1}{3}$$

$$(\sin x+\cos x)^2 = 1 + 2\left(-\frac{1}{3}\right) = \frac{1}{3} \Rightarrow \sin x+\cos x = -\frac{\sqrt{3}}{3}$$

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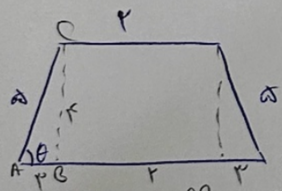

$$\rightarrow 3+3+2=8$$

$$S = \frac{(8+2)4}{2} = 20$$

$$\cos\theta = 0.6 = \frac{AB}{5} = \frac{6}{10} \Rightarrow AB = 3$$

$$\sin\theta = 0.8 = \frac{BC}{5} \Rightarrow BC = 4$$

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$$\tan(285^\circ)\tan(-165^\circ) - \sin(1095^\circ)\cos(255^\circ) = K\cos^2 15$$

$$\tan(270+15)\cdot(-\tan(180-15)) - \sin(3\times360+15)\cos(270-15)$$

$$(-\cot 15)(\tan 15) + \sin 15 \sin 15 = \sin^2 15 - 1 = -\cos^2 15^\circ \Rightarrow \boxed{K=-1}$$

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$$A=\frac{\sqrt{۳}\cos(۲۱۰^\circ)\overset{\overset{-\cos ۲۷}{\sin(۲۷۰-۲۷)}}{\sin(۲۴۳^\circ)}-\sqrt{۲}\sin(۱۳۵^\circ)\overset{\cos(۱۸۰-۲۷)}{\cos(۱۵۳^\circ)}}{\cos(۲۷^\circ)}=\frac{\sqrt{۳}\times\left(-\frac{\sqrt{۳}}{۲}\right)(-\cos۲۷^\circ)+\sqrt{۲}\times\frac{\sqrt{۲}}{۲}(-\cos۲۷^\circ)}{\cos۲۷^\circ}$$

$$\frac{\frac{۵}{۲}\cos۲۷^\circ}{\cos۲۷^\circ}=\boxed{\frac{۵}{۲}}$$

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$$f\left(\frac{\pi}{۲۴}\right)=۱۶\cos^۲(۲x)\cos^۲(۶x)\cos^۲(۱۲x)\cos^۲(۲۴x)$$

$$۱۶\cos^۲\left(\frac{\pi}{۱۲}\right)\cos^۲\left(\frac{\pi}{۴}\right)\cos^۲\left(\frac{\pi}{۳}\right)\cos^۲\left(\frac{۲\pi}{۳}\right)$$

$$۱۶\left(\frac{۲+\sqrt{۳}}{۴}\right)\left(\frac{\sqrt{۲}}{۲}\right)^۲\left(\frac{۱}{۲}\right)^۲\left(\frac{۱}{۲}\right)^۲$$

$$\cancel{۱۶}\left(\frac{۲+\sqrt{۳}}{\cancel{۴}}\right)\frac{۳}{\cancel{۴}}\quad\frac{۱}{۴}\quad\frac{۱}{۴}=\frac{۳(۲+\sqrt{۳})}{۱۶}$$

$$\cos^۲\alpha=\frac{۱+\cos۲\alpha}{۲}$$

$$\cos^۲\frac{\pi}{۱۲}=\frac{۱+\cos\left(\frac{\pi}{۶}\right)}{۲}=\frac{\frac{۲}{۲}+\frac{\sqrt{۳}}{۲}}{۲}=\frac{۲+\sqrt{۳}}{۴}$$

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راه دوم

$$\frac{۱-\sin x}{۱+\sin x}=۴\quad=۱-\sin x=۴+۴\sin x$$

$$-۵\sin x=۳$$

$$\sin x=-\frac{۳}{۵}\longrightarrow\cos x=-\frac{۴}{۵}$$

$$\sin^۲+\cos^۲=۱$$

$$\tan\frac{x}{۲}$$

$$\tan x=\frac{۲\tan\frac{x}{۲}}{۱-\tan^۲\frac{x}{۲}}=\frac{۳}{\cancel{۴}\,۸}\;\longrightarrow\;۳-۳\tan^۲\frac{x}{۲}=۸\tan\frac{x}{۲}\Longrightarrow ۳\tan^۲\frac{x}{۲}+۸\tan\frac{x}{۲}-۳=۰$$

$$۳t^۲+۸t-۳=۰$$

$$t^۲+۸t-۹=۰$$

$$(t+۹)(t-۱)=۰$$

$$-۳=-\frac{۹}{۳}\qquad\frac{۱}{۳}$$ غ ق ق

$$\boxed{\tan\frac{x}{۲}=-۳}\Longleftarrow$$

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$$\frac{\sin\theta}{۱-\cos\theta}+\frac{\overset{\sin^۲\theta}{(۱-\cos\theta)}(۱+\cos\theta)}{\sin\theta}=k\cot\frac{\theta}{۲}=\frac{\overset{۲\sin^۲\theta}{\sin^۲\theta+\sin^۲\theta}}{\sin\theta-\sin\theta\cos\theta}=\frac{۲\cancel{\sin^۲\theta}\,(۱+\cos\theta)}{\underbrace{(۱-\cos\theta)(۱+\cos\theta)}_{\cancel{\sin^۲\theta}\ \sin\theta}}$$

$$\frac{۲\overset{۲\cos^۲\frac{\theta}{۲}}{(۱+\cos\theta)}}{\sin\theta}=\frac{۴\cancel{\cos\frac{\theta}{۲}}\cos\frac{\theta}{۲}}{۲\sin\frac{\theta}{۲}\cancel{\cos\frac{\theta}{۲}}}=۲\cot\frac{\theta}{۲}\Rightarrow\boxed{k=۲}$$

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راه دوم

$$\sin\alpha=\frac{\sqrt{۲}}{۱۰}\longrightarrow\cos\alpha=\frac{-۷\sqrt{۲}}{۱۰}$$

$$\cos\left(\overset{۲\pi+\frac{۳\pi}{۴}+\alpha}{\frac{۱۱\pi}{۴}+\alpha}\right)=\cos\left(\frac{۳\pi}{۴}+\alpha\right)=\cos\frac{۳\pi}{۴}\cos\alpha-\sin\frac{۳\pi}{۴}\sin\alpha=\frac{-\sqrt{۲}}{۲}\times\left(\frac{-۷\sqrt{۲}}{۱۰}\right)-\left(\frac{\sqrt{۲}}{۲}\right)\left(\frac{\sqrt{۲}}{۱۰}\right)$$

$$\frac{۷}{۱۰}-\frac{۱}{۱۰}=\boxed{\frac{۶}{۱۰}}$$

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