روز الدو صدرالعلمائی — یازدهم ریاضی C — تکلیف ۲۸

۱ـ

$\Rightarrow |\sin\alpha|$ چون صورت و مخرج مثبت هستند $\Rightarrow$ باید مثبت باشد $\Rightarrow \sin\alpha > 0$

$$\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} \quad , \quad \frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|}$$

($\sqrt{1-\cos^2\alpha}$: $\sin^2\alpha$)

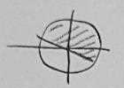
$\Rightarrow$ ربع اول

$$\cot\alpha = \frac{\cos\alpha}{|\sin\alpha|} \qquad \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{\cos\alpha} \Rightarrow \cos\alpha > 0$$

۲ـ

$$\sin 2x = \frac{m-1}{4} \qquad -\frac{\pi}{6} < 2x < \frac{5\pi}{6} \Rightarrow -\frac{1}{2} < \sin 2x \leq 1$$

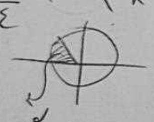
$$-\frac{\pi}{12} < x < \frac{5\pi}{12}$$



$$-2 < m-1 \leq 4 \rightarrow \boxed{-1 < m \leq 5}$$

۳ـ

$$\tan x + \cot x = -3$$
$$3\pi < 4x < 4\pi$$
$$\frac{3\pi}{4} < x < \pi$$



اندازه $\cos\alpha$ بزرگتر از $\sin\alpha$

$$\frac{1}{\sin^3 x + \cos^3 x} = \frac{1}{(\sin x + \cos x)(\sin^2 + \cos^2 - \sin x\cos x)} = \frac{9}{-4\sqrt{3}}$$

($\sin x + \cos x = -\frac{\sqrt{3}}{3}$, $1 + \frac{1}{3} = \frac{4}{3}$)

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{1}{\sin x\cos x} = -3 \quad \sin x\cos x = -\frac{1}{3}$$

$$(\sin x + \cos x)^2 = 1 + 2\left(-\frac{1}{3}\right) = \frac{1}{3} \Rightarrow \sin x + \cos x = -\frac{\sqrt{3}}{3}$$

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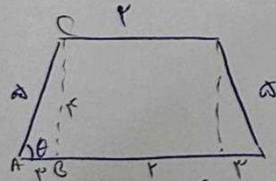

$$\rightarrow 3+3+2 = 8$$

$$S = \frac{(8+2)\,4}{2} = \boxed{20}$$

$$\sin\theta = 0.8 = \frac{BC}{5} \Rightarrow BC = 4$$

$$\cos\theta = 0.6 = \frac{AB}{5} = \frac{6}{10} \Rightarrow AB = 3$$

۵ـ

$$\tan(285^\circ)\tan(-165^\circ) - \sin(1095^\circ)\cos(255^\circ) = k\cos^2 15$$

$$\tan(270+15)\cdot(-\tan(180-15)) - \sin(3\times 360+15)\cos(270-15)$$

$$(-\cot 15)(\tan 15) + \sin 15 \sin 15 = \sin^2 15 - 1 = -\cos^2 15^\circ \Rightarrow \boxed{k=-1}$$

-٩

$$A=\frac{\sqrt{2}\cos(210^\circ)\sin(225^\circ)-\sqrt{2}\sin(135^\circ)\cos(135^\circ)}{\cos(45^\circ)}=\frac{\sqrt{2}\times\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)+\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$$

$$\frac{\frac{\sqrt{6}}{4}+\frac{1}{2}}{\frac{\sqrt{2}}{2}}=\frac{\frac{\sqrt{6}+2}{4}}{\frac{\sqrt{2}}{2}}=\frac{\sqrt{6}+2}{2\sqrt{2}}=\frac{\sqrt{3}+\sqrt{2}}{2}$$

-٧

$$f(x)=14\cos^2(3x)\cos^2(4x)\cos^2(12x)\cos^2(24x)$$

$$\cos^2\alpha=\frac{1+\cos2\alpha}{2}$$

$$14\cos^2\left(\frac{\pi}{12}\right)\cos^2\left(\frac{\pi}{6}\right)\cos^2\left(\frac{\pi}{3}\right)\cos^2\left(\frac{2\pi}{3}\right)$$

$$\cos^2\frac{\pi}{12}=\frac{1+\cos\left(\frac{\pi}{6}\right)}{2}=\frac{\frac{2}{2}+\frac{\sqrt{3}}{2}}{2}=\frac{2+\sqrt{3}}{4}$$

$$14\left(\frac{2+\sqrt{3}}{4}\right)\left(\frac{\sqrt{3}}{2}\right)^2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2=14\left(\frac{2+\sqrt{3}}{4}\right)\frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}=\frac{3(2+\sqrt{3})}{14}$$

-٨

$$\frac{1-\sin\alpha}{1+\sin\alpha}=4\Rightarrow 1-\sin\alpha=4+4\sin\alpha$$

$$-5\sin\alpha=3$$

$$\sin\alpha=-\frac{3}{5}\longrightarrow\cos\alpha=-\frac{4}{5}$$

$$\sin^2+\cos^2=1$$

$$\tan\frac{\alpha}{2}$$

$$\tan\alpha=\frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}}=\frac{3}{4}\Rightarrow 3-3\tan^2\frac{\alpha}{2}=8\tan\frac{\alpha}{2}\Longrightarrow 3\tan^2\frac{\alpha}{2}+8\tan\frac{\alpha}{2}-3=0$$

$$3t^2+8t-3=0$$

$$t^2+8t-9=0$$

$$(t+9)(t-1)=0$$

$$t=1,\ t=-9$$

$$\tan\frac{\alpha}{2}=-3$$

-٩

$$\frac{\sin\theta}{1-\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=k\cot\frac{\theta}{2}$$

$$\frac{\sin^2\theta+(1+\cos\theta)^2}{\sin\theta(1-\cos\theta)}=\frac{2\sin^2\theta}{\sin\theta-\sin\theta\cos\theta}=\frac{2\sin^2\theta(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)\sin\theta}$$

$$\frac{2(1+\cos\theta)}{\sin\theta}=\frac{4\cos^2\frac{\theta}{2}\cos\frac{\theta}{2}}{4\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{\theta}{2}}=2\cot\frac{\theta}{2}=k\Rightarrow k=2$$

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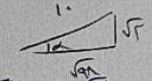

$$\sin\alpha=\frac{\sqrt{2}}{10}\longrightarrow\cos\alpha=\frac{-\sqrt{98}}{10}$$

$$\frac{7}{10}-\frac{1}{10}=\frac{6}{10}$$

$$\cos\left(\frac{11\pi}{4}+\frac{2\pi}{4}+\alpha\right)=\cos\left(\frac{3\pi}{4}+\alpha\right)=\cos\frac{3\pi}{4}\cos\alpha-\sin\frac{3\pi}{4}\sin\alpha=-\frac{\sqrt{2}}{2}\times\left(\frac{-7\sqrt{2}}{10}\right)-\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{10}\right)$$