19,70

-۱

$\cot\alpha = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow \frac{\cos\alpha}{|\sin\alpha|} = \frac{\cos\alpha}{\sin\alpha} \Rightarrow |\sin\alpha| = \sin\alpha \rightarrow \sin\alpha > 0$ ربع اول

$\frac{\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha}}{\frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha}} = \frac{1-\sin\alpha}{|\cos\alpha|} \Rightarrow$ چون مخرج مشترک بین $\cos\alpha$ و $|\cos\alpha|$ تغییری هم در $|\cos\alpha|$ شده علامت صورت ایجاد نشده پس $|\cos\alpha| = \cos\alpha$ $\Rightarrow \cos\alpha > 0$

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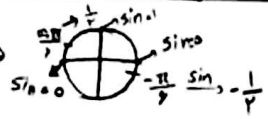
-۲

$-\frac{\pi}{6} < 2\alpha < \frac{5\pi}{6} \Rightarrow$ 

$-\frac{1}{2} < \underbrace{\sin 2\alpha}_{m-1} \leq 1 \Rightarrow -1 < m \leq 2$

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-۳

$\frac{3\pi}{4} < \alpha < \pi \rightarrow |\cos\alpha| > |\sin\alpha|$ ، $\tan\alpha + \cot\alpha = \frac{1}{\sin\alpha\cos\alpha} = -3 \Rightarrow \sin\alpha\cos\alpha = -\frac{1}{3}$

$(-\sin\alpha + \cos\alpha)^2 = \sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha$

$(\sin\alpha + \cos\alpha)^2 = 1 - \frac{2}{3} = \frac{1}{3}$

$|\sin\alpha + \cos\alpha| = \frac{\sqrt{3}}{3} \Rightarrow \sin\alpha + \cos\alpha = -\frac{\sqrt{3}}{3}$

$\frac{1}{\underbrace{(\sin\alpha+\cos\alpha)}_{-\frac{\sqrt{3}}{3}}\left(\underbrace{\sin^2\alpha+\cos^2\alpha}_{1} \underbrace{-\sin\alpha\cos\alpha}_{+\frac{1}{3}}\right)} = \frac{9}{-4\sqrt{3}} = \frac{-3\sqrt{3}}{4}$

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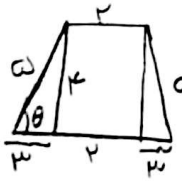
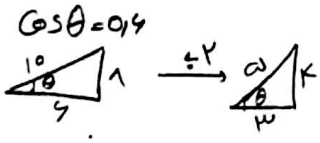


$\cos\theta = 0.6$



$\Rightarrow S = \frac{(2+1)4}{2} = 10$

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$\underbrace{\tan(270^\circ+15^\circ)}_{-\cot 15^\circ}\underbrace{\tan(-110^\circ+15^\circ)}_{\tan 15^\circ} - \underbrace{\sin(3\times 360^\circ+15^\circ)}_{\sin 15^\circ}\underbrace{\cos(270^\circ-15^\circ)}_{-\sin 15^\circ} = -1+\sin^2 15^\circ = -\cos^2 15^\circ$

$\Rightarrow -\cos^2 15^\circ = K\cos^2 15^\circ \rightarrow K = -1$

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$A = \sqrt{3}\underbrace{\cos(210^\circ)}_{-\frac{\sqrt{3}}{2}}\underbrace{\sin(270^\circ-27^\circ)}_{-\cos 27^\circ} - \sqrt{2}\underbrace{\sin(135^\circ)}_{\frac{\sqrt{2}}{2}}\underbrace{\cos(180^\circ-27^\circ)}_{-\cos 27^\circ} = \frac{3}{2}\cos 27^\circ$

$\frac{\frac{3}{2}\cos 27^\circ}{\cos 27^\circ} = \frac{3}{2}$

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-۷

$f\left(\frac{\pi}{24}\right) = 16\cos^2\left(\frac{2\pi}{24}\right)\underbrace{\cos^2\left(\frac{4\pi}{24}\right)}_{\frac{3}{4}}\underbrace{\cos^2\left(\frac{12\pi}{24}\right)}_{\frac{1}{4}}\underbrace{\cos^2\left(\frac{24\pi}{24}\right)}_{\frac{1}{4}} = 16\times\left(\frac{2+\sqrt{3}}{4}\right)\times\frac{3}{4^3} = \frac{6+3\sqrt{3}}{16}$

$\cos^2\frac{\pi}{12} = \frac{1+\cos\frac{\pi}{6}}{2} = \frac{1+\frac{\sqrt{3}}{2}}{2} = \frac{2+\sqrt{3}}{4}$

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-۸

$$\div \cos\alpha \rightarrow \frac{\frac{1}{\cos\alpha} - \tan\alpha}{\frac{1}{\cos\alpha} + \tan\alpha} = 4 \Rightarrow 4\frac{1}{\cos\alpha} + 4\tan\alpha = \frac{1}{\cos\alpha} - \tan\alpha$$

$$3\tan\alpha = -3\frac{1}{\cos\alpha} \quad \text{تا توان ۲}$$

$$9\tan^2\alpha = 9(1+\tan^2\alpha) \Rightarrow 16\tan^2\alpha = 9 \xrightarrow[\tan\alpha<0]{\text{ربع سوم}} \tan\alpha = -\frac{3}{4}$$

$$\tan\alpha = \frac{2\tan(\frac{\alpha}{2})}{1-\tan^2(\frac{\alpha}{2})} \xrightarrow[\tan(2\alpha)=\frac{2\tan\alpha}{1-\tan^2\alpha}]{\text{طبق این فرمول}} 3 - 3\tan^2(\frac{\alpha}{2}) = 8\tan(\frac{\alpha}{2}) \Longrightarrow$$

$$3\tan^2(\frac{\alpha}{2}) + 8\tan(\frac{\alpha}{2}) - 3 = 0 \xrightarrow{\text{تجزیه}} (\tan(\frac{\alpha}{2})+9)(\tan(\frac{\alpha}{2})-1) = 0$$

$$\tan(\frac{\alpha}{2}) = \frac{-9}{3} = -3 \quad \text{غ ق ق}$$

$$\tan(\frac{\alpha}{2}) = \frac{1}{3}$$

-۹

$$\frac{\sin\theta}{1-\cos\theta} + \frac{1-\cos\theta}{\sin\theta} = \frac{2\sin^2\theta}{\sin\theta(1-\cos\theta)} = \frac{2\sin\theta}{1-\cos\theta} = \frac{2\sin(\frac{\theta}{2})\cos(\frac{\theta}{2})}{\sin^2(\frac{\theta}{2})} = 2\frac{\cos(\frac{\theta}{2})}{\sin(\frac{\theta}{2})}$$

$$\Rightarrow 2\cot(\frac{\theta}{2}) = k\cot(\frac{\theta}{2}) \longrightarrow k = 2$$

-۱۰

$$\cos\alpha = -\sqrt{1-\frac{2}{100}} = -\frac{\sqrt{98}}{10}$$

$$\cos(\frac{11\pi}{4}+\alpha) = \underbrace{\cos\frac{11\pi}{4}}_{-\frac{\sqrt{2}}{2}}\underbrace{\cos\alpha}_{-\frac{\sqrt{98}}{10}} - \underbrace{\sin\frac{11\pi}{4}}_{\frac{\sqrt{2}}{2}}\underbrace{\sin\alpha}_{\frac{\sqrt{2}}{10}} = \frac{14}{20} - \frac{1}{10} = \frac{6}{10} = \frac{3}{5}$$