تمرین تکلیف ۲۸ — ستایش منصور

مقطع: یازدهم تجربی (دفتر)

۳ خرداد ۱۴۰۰

سؤال ۱:

$\cot a = \frac{\cos a}{\sqrt{1-\cos^2 a}}$, $\frac{1}{\sqrt{\cos^2 a}} - \frac{1}{\cot a} = \frac{1-\sin a}{|\cos a|}$ انتهای کمان a = ?

$\sin a > 0$ ، $|\sin a|$

$\frac{1}{|\cos a|} - \frac{\sin a}{\cos a} = \frac{1-\sin a}{|\cos a|} \Rightarrow \cos a > 0$ } ⇒ انتهای کمان a ⇒ ربع اول

سؤال ۲:

$\sin 2x = \frac{m-1}{4}$, $-\frac{\pi}{12} < x < \frac{5\pi}{12}$ مجموع مقادیر m = ?

$-\frac{\pi}{12} < x < \frac{5\pi}{12} \to -\frac{\pi}{6} < 2x < \frac{5\pi}{6} \to -\frac{1}{2} < \sin 2x \leq 1$

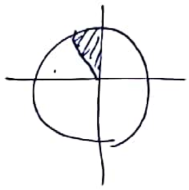
$\Rightarrow -\frac{1}{2} < \frac{m-1}{4} \leq 1 \to -2 < m-1 \leq 4 \to -1 < m \leq 5 \Rightarrow (-1, 5]$

سؤال ۳:

$\tan x + \cot x = -3$, $3\pi < 4x < 4\pi$ $\frac{1}{\sin^3 x + \cos^3 x} = ?$

$\tan x + \cot x = -3 \to \frac{2}{\sin 2x} = -3 \to -\frac{1}{3} = \sin x \cos x$

$3\pi < 4x < 4\pi \to \frac{3\pi}{4} < x < \pi$



$\frac{1}{\sin^3 x + \cos^3 x} = \frac{1}{(\sin x + \cos x)(1 - \sin x \cos x)} = \frac{1}{\frac{4}{3\sqrt{3}}} = \frac{3\sqrt{3}}{4}$

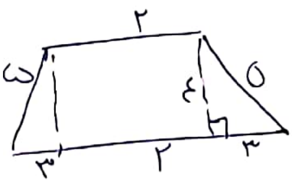
($\sin x + \cos x = \frac{1}{\sqrt{3}}$, $1 - \sin x \cos x = \frac{4}{3}$)

سؤال ۴:

زاویه حاده بین ساق و قاعده، θ

قاعده کوچک = ۲ ، ساق = ۵ ، $\cos\theta = 0{,}6$

مساحت ذوزنقه = ?



$S = \frac{(2+8) \times 4}{2} = 20$

سؤال ۵:

$\tan(285^\circ)\tan(-165^\circ) - \sin(105^\circ)\cos(255^\circ) = k\cos^2 15$ $k = ?$

$(\tan(285)\tan 15) - \sin 15 \times \sin 15 \Rightarrow -1 + \sin^2 15 = -\cos^2 15$

($\tan(285) = -\cot 15$)

$-\cos^2 15 = k\cos^2 15 \to k = -1$

س ۲،

$A = \sqrt{3}\cos(210^\circ)\sin(243^\circ) - \sqrt{2}\sin(135^\circ)\cos(153^\circ) = ?\cos(27^\circ)$

$\underbrace{\sqrt{3}\cos(210^\circ)}_{-\frac{3}{2}}\ \sin(\underbrace{243^\circ}_{\frac{3\pi}{2}-27}) - \sqrt{2}\sin(135^\circ)\cos(\underbrace{153^\circ}_{\pi-27}) =$

$\to \frac{3}{2}\cos 27 + \cos 27 = \frac{5}{2}\cos 27^\circ \Rightarrow$ ۵/۲ برابر

س ۷:

$f(x) = 16\cos^2(3x)\cos^2(6x)\cos^2(12x)\cos^2(24x)$ در $\frac{\pi}{36}$

$16\cos^2(\frac{\pi}{12})\cos^2(\frac{\pi}{6})\cos^2(\frac{\pi}{3})\cos^2(\frac{2\pi}{3}) = (16 \times \frac{2+\sqrt{3}}{4})(\frac{3}{4})(\frac{1}{4})(\frac{1}{4}) = \frac{3(2+\sqrt{3})}{16}$

س ۸

اگر انتهای کمان x : $\frac{1-\sin x}{1+\sin x} = 4$ $\quad \tan\frac{x}{2} = ?$ $\quad \tan\begin{cases}\frac{1}{3} \\ \boxed{-3}\end{cases} \leftarrow \tan\frac{x}{2} = 1, -9$

$1 - \sin x = 4 + 4\sin x$

$5\sin x = -3 \to \sin x = -\frac{3}{5} \to \cos x = -\frac{4}{5}$

$\tan x = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}} \to \frac{3}{4} = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}} \to 3 - 3\tan^2\frac{x}{2} = 8\tan\frac{x}{2} \to 3\tan^2\frac{x}{2} + 8\tan\frac{x}{2} - 3 = 0$

$\tan^2\frac{x}{2} + 8\tan\frac{x}{2} - 9 = 0$

س ۹،

$\frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = k\cot\frac{\theta}{2}$ $\quad k = ?$

$\frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = \frac{\sin^2\theta + 1 - \cos^2\theta}{\sin\theta(1-\cos\theta)} = \frac{2\sin\theta(1+\cos\theta)}{\underbrace{1-\cos^2\theta}_{\sin^2\theta}} = \frac{2(1+\cos\theta)}{\sin\theta} = 2\cot\frac{\theta}{2}$

$2\cot\frac{\theta}{2} = k\cot\frac{\theta}{2} \Rightarrow \boxed{k = 2}$

س ۱۰

اگر انتهای کمان a : $\sin a = \frac{\sqrt{2}}{10}$ $\quad \cos(\frac{11\pi}{4} + a) = ?$

$\cos a = -\frac{7}{5\sqrt{2}}$

$\cos(\frac{11\pi}{4} + a) = -\frac{\sqrt{2}}{2}\cos a - \frac{\sqrt{2}}{2}\sin a = -\frac{\sqrt{2}}{2}(\cos a + \sin a) = -\frac{\sqrt{2}}{2}(\frac{-7}{5\sqrt{2}} + \frac{1}{5\sqrt{2}}) = \boxed{\frac{6}{10}}$