ژانت آقایی — تکلیف شماره ۲۸ — یازدهم تجربی

۱۷٫۲۵

.1

$$\cot\alpha=\frac{\cos\alpha}{\sqrt{\sin^2\alpha}}=\frac{\cos\alpha}{|\sin\alpha|}=\frac{\cos\alpha}{\sin\alpha}\quad \sin\alpha>0$$

$$\frac{1}{|\cos\alpha|}-\frac{1}{\cot\alpha}=\frac{1-\sin\alpha}{|\cos\alpha|}$$

$$\frac{1}{|\cos\alpha|}-\frac{|\sin\alpha|}{\cos\alpha}=\frac{1-\sin\alpha}{|\cos\alpha|}$$

$\cos\alpha>0,\ \sin\alpha>0:\quad \frac{1}{\cos\alpha}-\frac{\sin\alpha}{\cos\alpha}=\frac{1-\sin\alpha}{\cos\alpha}$ ✓

۵

$\cos\alpha<0,\ \sin\alpha>0:\quad \frac{-1-\sin\alpha}{\cos\alpha}=\frac{1-\sin\alpha}{-\cos\alpha}$ ×

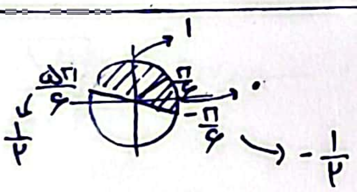
$\cos\alpha<0,\ \sin\alpha<0:\quad \frac{-1+\sin\alpha}{\cos\alpha}=\frac{1-\sin\alpha}{-\cos\alpha}$ ✓ $\to \sin>0$ غ

$\cos\alpha>0,\ \sin\alpha<0:\quad \frac{1+\sin\alpha}{\cos\alpha}=\frac{1-\sin\alpha}{\cos\alpha}$ ×

ناحیه‌ی اول

.2

$$-\frac{\pi}{6}<2x<\frac{5\pi}{6}$$



$(-1,5]$

۵

$$-\frac{1}{2}<\sin 2x\leqslant 1\to -\frac{1}{2}<\frac{m-1}{4}\leqslant 1\to -2<m-1\leqslant 4\to -1<m\leqslant 5$$

.3

$$\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}=\frac{\overbrace{\sin^2x+\cos^2x}^{1}}{\sin x\cos x}=-3$$

$$\sin x\cos x=-\frac{1}{3}$$

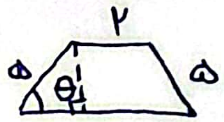
۵

$$\frac{1}{(\sin x+\cos x)(\underbrace{\sin^2x+\cos^2x}_{1}-\underbrace{\sin x\cos x}_{-\frac{1}{3}})}$$

$$(\sin x+\cos x)^2=\underbrace{\sin^2x+\cos^2x}_{1}+\underbrace{2\sin x\cos x}_{-\frac{2}{3}}=\frac{1}{3}\to \sin x+\cos x=\pm\sqrt{\frac{1}{3}}$$

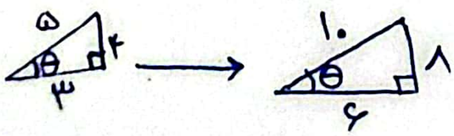
$$\frac{1}{\frac{4}{3}\times\sqrt{\frac{1}{3}}}=\ominus\frac{3\sqrt{3}}{4}$$

.4



$$S=\frac{(2+8)\times 4}{2}=20$$

۵



$$\underbrace{\tan(۲۷۰+۱۵)}_{-\cot ۱۵}\ \underbrace{\tan(-۱۸۰+۱۵)}_{+\tan ۱۵}-\underbrace{\sin(۱۰۸۰+۱۵)}_{+\sin ۱۵}\ \underbrace{\cos(۲۷۰-۱۵)}_{-\sin ۱۵}=$$ .5

$$\underbrace{\underbrace{-\tan ۱۵\ \cot ۱۵}_{-۱}+\sin^{۲} ۱۵}_{-\cos^{۲} ۱۵}=k\cos^{۲} ۱۵ \longrightarrow k=-۱$$

.6

$$\sqrt{۳}\,\underbrace{\cos ۲۱۰^\circ}_{\frac{\sqrt{۳}}{۲}}\,\sin(۲۷۰-۲۷)-\sqrt{۲}\,\underbrace{\sin(۱۳۵)}_{\frac{\sqrt{۲}}{۲}}\,\cos(۱۸۰-۲۷)$$

$$+\frac{۳}{۲}(\cos ۲۷)+\cos ۲۷=\frac{\frac{۵}{۲}\cos ۲۷}{\cos ۲۷}=\frac{۵}{۲}$$

۵

.7

$$\cancel{۱۶}\cos^۲\left(\frac{\pi}{۱۲}\right)\underbrace{\cos^۲\left(\frac{\pi}{۶}\right)}_{\frac{۳}{\cancel{۴}}}\underbrace{\cos^۲\left(\frac{\pi}{۳}\right)}_{\frac{۱}{\cancel{۴}}}\underbrace{\cos^۲\left(\frac{۲\pi}{۳}\right)}_{\frac{۱}{۴}}=\frac{۳+۳\sqrt{۳}}{۱۶}$$

$$\cos^۲\frac{\pi}{۱۲}=\frac{۱+\cos\frac{\pi}{۶}}{۲}=\frac{۱+\frac{\sqrt{۳}}{۲}}{۲}=\frac{۱+\sqrt{۳}}{۴}$$

۱٫۵

.8

$$\frac{۱-\sin x}{۱+\sin x}\xrightarrow{\div\cos}\frac{\frac{۱}{\cos x}-\tan x}{\frac{۱}{\cos x}+\tan x}=۴\longrightarrow ۴\frac{۱}{\cos x}+۴\tan x=\frac{۱}{\cos x}-\tan x\longrightarrow$$

ربع سوم

$$۵\tan x=-۳\frac{۱}{\cos x}\xrightarrow{(\ )^۲}۲۵\tan^۲ x=۹(۱+\tan^۲ x)\longrightarrow ۱۶\tan^۲ x=۹\longrightarrow \tan x=\frac{۳}{۴}$$

۱٫۷۵

$$\underbrace{\cancel{\tan x}}_{\frac{۳}{۴}}=\frac{۲\tan(\frac{x}{۲})}{۱-\tan^۲(\frac{x}{۲})}\longrightarrow ۸\tan\left(\frac{x}{۲}\right)=۳-۳\tan^۲\left(\frac{x}{۲}\right)\longrightarrow ۳\tan^۲\left(\frac{x}{۲}\right)+۸\tan\left(\frac{x}{۲}\right)-۳$$

غ ق ق

$$۳t^۲+۸t-۳=۰\longrightarrow t^۲+۸t-۹=۰\longrightarrow t=۱,\ t=-۹\longrightarrow \tan\left(\frac{x}{۲}\right)=\cancel{\boxed{\frac{۱}{۳}}},\ -۳$$

.9

$$\frac{\sin\theta}{۱-\cos\theta}+\frac{۱+\cos\theta}{\sin\theta}=\frac{\sin^۲\theta+\sin^۲\theta}{\sin\theta(۱-\cos\theta)}=\frac{۲\sin\theta}{۱-\cos\theta}=\frac{۲\sin(\frac{\theta}{۲})\cos(\frac{\theta}{۲})}{۲\sin^۲(\frac{\theta}{۲})}=\frac{\cos(\frac{\theta}{۲})}{\sin(\frac{\theta}{۲})}$$

۱

$$\frac{\cos(\frac{\theta}{۲})}{\sin(\frac{\theta}{۲})}=k\cot\left(\frac{\theta}{۲}\right)\longrightarrow\boxed{k=۱}$$

۹) $\frac{\sin^۲\theta+(۱-\cos^۲\theta)}{(۱-\cos\theta)\sin\theta}=\frac{۲\sin^۲\theta}{\sin\theta(۱-\cos\theta)}=\frac{۲\times۲\times\sin\frac{\theta}{۲}\cos\frac{\theta}{۲}}{۲\sin^۲\frac{\theta}{۲}}=۲\cot\frac{\theta}{۲}$

$\to k=۲$

CamScanner

$$\cos\left(\frac{11\pi}{4}+\alpha\right)=\underbrace{\cos\frac{11\pi}{4}}_{-\frac{\sqrt{2}}{2}}\;\underbrace{\cos\alpha}_{-\frac{\sqrt{98}}{10}}-\underbrace{\sin\frac{11\pi}{4}}_{-\frac{\sqrt{2}}{2}}\;\underbrace{\sin\alpha}_{\frac{\sqrt{2}}{10}}=\frac{14}{20}+\frac{2}{20}=\frac{16}{20}=\boxed{0/8}$$

$$\sin^2\alpha+\cos^2\alpha=1\longrightarrow\frac{2}{100}+\frac{98}{100}=1\longrightarrow\cos\alpha=\sqrt{\frac{98}{100}}=-\frac{\sqrt{98}}{10}$$

$$\cos\left(\frac{11\pi}{4}+\alpha\right)=-\left(\cos\alpha\cos\frac{\pi}{4}+\sin\alpha\sin\frac{\pi}{4}\right)$$

$$\to\frac{-\sqrt{2}}{2}(\cos\alpha+\sin\alpha)\qquad\cos\alpha=\frac{-7\sqrt{2}}{10}$$

$$\hookrightarrow\frac{-\sqrt{2}}{2}\left(\frac{-7\sqrt{2}}{10}+\frac{\sqrt{2}}{10}\right)=\frac{3}{5}$$

7) $f\left(\frac{\pi}{24}\right)=16\cos^2\left(\frac{\pi}{12}\right)\cos^2\left(\frac{\pi}{6}\right)\cos^2\left(\frac{\pi}{3}\right)\cos^2\left(\frac{2\pi}{3}\right)$

$$\cos^2\frac{\pi}{12}=\frac{1+\cos\frac{\pi}{6}}{2}=\frac{2+\sqrt{3}}{4}$$

$$16\left(\frac{2+\sqrt{3}}{4}\right)\times\frac{3}{4}\times\frac{1}{4}\times\frac{1}{4}=\frac{3(2+\sqrt{3})}{16}$$