.1

$$\cot\alpha = \frac{\cos\alpha}{\sqrt{\sin^2\alpha}} = \frac{\cos\alpha}{|\sin\alpha|} = \frac{\cos\alpha}{\sin\alpha} \quad \sin\alpha>0$$

$$\frac{1}{|\cos\alpha|} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|}$$

$$\frac{1}{|\cos\alpha|} - \frac{|\sin\alpha|}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|}$$

$\cos\alpha>0,\ \sin\alpha>0$: $\quad \dfrac{1}{\cos\alpha} - \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{1-\sin\alpha}{\cos\alpha}$ ✓

$\cos\alpha<0,\ \sin\alpha>0$: $\quad \dfrac{-1-\sin\alpha}{\cos\alpha} = \dfrac{1-\sin\alpha}{-\cos\alpha}$ ×

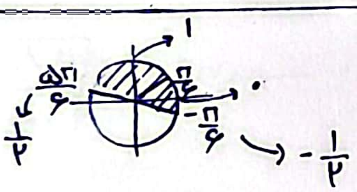
$\cos\alpha<0,\ \sin\alpha<0$: $\quad \dfrac{-1+\sin\alpha}{\cos\alpha} = \dfrac{1-\sin\alpha}{-\cos\alpha}$ ✓ $\to \sin>0$ غ

$\cos\alpha>0,\ \sin\alpha<0$: $\quad \dfrac{1+\sin\alpha}{\cos\alpha} = \dfrac{1-\sin\alpha}{\cos\alpha}$ ×

ناحیه‌ی اول

.2

$$-\frac{\pi}{6} < 2x < \frac{5\pi}{6}$$



$(-1, 5]$

$$-\frac{1}{2} < \sin 2x \leq 1 \to -\frac{1}{2} < \frac{m-1}{4} \leq 1 \to -2 < m-1 \leq 4 \to -1 < m \leq 5$$

.3

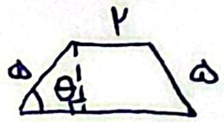
$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\overbrace{\sin^2 x + \cos^2 x}^{1}}{\sin x\cos x} = -3 \to \sin x\cos x = -\frac{1}{3}$$

$$\frac{1}{(\sin x+\cos x)(\underbrace{\sin^2 x+\cos^2 x}_{1} - \underbrace{\sin x\cos x}_{-\frac{1}{3}})}$$

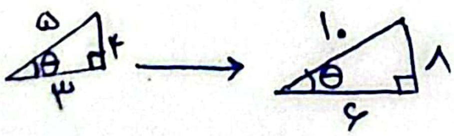
$$(\sin x+\cos x)^2 = \underbrace{\sin^2 x + \cos^2 x}_{1} + \underbrace{2\sin x\cos x}_{-\frac{2}{3}} = \frac{1}{3} \to \sin x+\cos x = \pm\sqrt{\frac{1}{3}}$$

$$\frac{1}{\frac{4}{3}\times\sqrt{\frac{1}{3}}} = -\frac{3\sqrt{3}}{4}$$

.4



$$S = \frac{(2+8)\times 4}{2} = 20$$



5.

$$\underbrace{\tan(270+15)}_{-\cot 15}\ \underbrace{\tan(-180+15)}_{+\tan 15} - \underbrace{\sin(1080+15)}_{+\sin 15}\ \underbrace{\cos(270-15)}_{-\sin 15} =$$

$$\underbrace{\underbrace{-\tan 15\ \cot 15}_{-1} + \sin^2 15}_{-\cos^2 15} = k\cos^2 15 \longrightarrow k = -1$$

6.

$$\sqrt{3}\underbrace{\cos 210^\circ}_{\frac{\sqrt{3}}{2}}\sin(270-27)-\sqrt{2}\underbrace{\sin(135)}_{\frac{\sqrt{2}}{2}}\cos(180-27)$$

$$+\frac{3}{2}(\cos 27)+\cos 27=\frac{\frac{5}{2}\cos 27}{\cos 27}=\frac{5}{2}$$

7.

$$\cancel{16}\underbrace{\cos^2\left(\frac{\pi}{12}\right)}\underbrace{\cos^2\left(\frac{\pi}{6}\right)}_{\frac{3}{\cancel{4}}}\underbrace{\cos^2\left(\frac{\pi}{3}\right)}_{\frac{1}{\cancel{4}}}\underbrace{\cos^2\left(\frac{2\pi}{3}\right)}_{\frac{1}{4}}=\frac{3+3\sqrt{3}}{16}$$

$$\cos^2\frac{\pi}{12}=\frac{1+\cos\frac{\pi}{6}}{2}=\frac{1+\frac{\sqrt{3}}{2}}{2}=\frac{1+\sqrt{3}}{4}$$

8.

$$\frac{1-\sin x}{1+\sin x}\xrightarrow{\div\cos}\frac{\frac{1}{\cos x}-\tan x}{\frac{1}{\cos x}+\tan x}=4\longrightarrow 4\frac{1}{\cos x}+4\tan x=\frac{1}{\cos x}-\tan x\longrightarrow$$

$$5\tan x=-3\frac{1}{\cos x}\xrightarrow{(\ )^2}25\tan^2 x=9(1+\tan^2 x)\longrightarrow 16\tan^2 x=9\xrightarrow{\text{ربع سوم}}\tan x=\frac{3}{4}$$

$$\underset{\frac{3}{4}}{\cancel{\tan x}}=\frac{2\tan(\frac{x}{2})}{1-\tan^2(\frac{x}{2})}\longrightarrow 8\tan\left(\frac{x}{2}\right)=3-3\tan^2\left(\frac{x}{2}\right)\longrightarrow 3\tan^2\left(\frac{x}{2}\right)+8\tan\left(\frac{x}{2}\right)-3$$

$$3t^2+8t-3=0\longrightarrow t^2+8t-9=0\longrightarrow t=1\ ,t=-9\longrightarrow \tan\left(\frac{x}{2}\right)=\boxed{\frac{1}{3}},\ \overset{\text{غ ق ق}}{-3}$$

9.

$$\frac{\sin\theta}{1-\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=\frac{\sin^2\theta+\sin^2\theta}{\sin\theta(1-\cos\theta)}=\frac{2\sin\theta}{1-\cos\theta}=\frac{2\sin(\frac{\theta}{2})\cos(\frac{\theta}{2})}{2\sin^2(\frac{\theta}{2})}=\frac{\cos(\frac{\theta}{2})}{\sin(\frac{\theta}{2})}$$

$$\frac{\cos(\frac{\theta}{2})}{\sin(\frac{\theta}{2})}=k\cot\left(\frac{\theta}{2}\right)\longrightarrow\boxed{k=1}$$

$$\cos\left(\frac{۱۱\pi}{۴}+\alpha\right)=\underbrace{\cos\frac{۱۱\pi}{۴}\,\cos\alpha}_{-\frac{\sqrt{۲}}{۲}\times-\frac{\sqrt{۹۸}}{۱۰}}-\underbrace{\sin\frac{۱۱\pi}{۴}\,\sin\alpha}_{-\frac{\sqrt{۲}}{۲}\times\frac{\sqrt{۲}}{۱۰}}=\frac{۱۴}{۲۰}+\frac{۲}{۲۰}=\frac{۱۶}{۲۰}=\boxed{۰/۸}$$

$$\sin^{۲}\alpha+\cos^{۲}\alpha=۱\longrightarrow\frac{۲}{۱۰۰}+\cancel{x}\,\frac{۹۸}{۱۰۰}=۱\longrightarrow\cos\alpha=\sqrt{\frac{۹۸}{۱۰۰}}=-\frac{\sqrt{۹۸}}{۱۰}$$