هستی رمضانی / یازدهم دفتر C

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۱)

$$\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} \qquad \frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|}$$

$$\cot\alpha = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow \sin\alpha > 0 \qquad \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \rightarrow \cos\alpha > 0 \Rightarrow$$ ناحیه اول

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۲)

$$-\frac{\pi}{12} < x < \frac{5\pi}{12} \longrightarrow -\frac{\pi}{6} < 2x < \frac{5\pi}{6} \longrightarrow -\frac{1}{2} < \sin 2x \leq 1$$

$$-\frac{1}{2} < \frac{m-1}{4} \leq 1 \longrightarrow -2 < m-1 \leq 4 \longrightarrow -1 < m \leq 5$$

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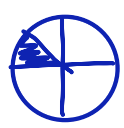
۳)

$$\sin^3 x + \cos^3 x = (\sin x + \cos x)(1 - \sin x\cos x) \longrightarrow \frac{1}{\sin^3 x + \cos^3 x} = \frac{1}{(\sin x + \cos x)(1-\sin x\cos x)} = \frac{1}{-\frac{1}{\sqrt{3}} \times \frac{4}{3}} = -\frac{3\sqrt{3}}{4}$$

$$\tan x + \cot x = \frac{1}{\sin x\cos x} = -3 \longrightarrow \sin x\cos x = -\frac{1}{3}$$

$$(\sin x + \cos x)^2 = 1 + 2\sin x\cos x = \frac{1}{3} \longrightarrow$$  $$\Rightarrow \sin x + \cos x < 0 \Rightarrow \sin x + \cos x = -\frac{1}{\sqrt{3}} \,,\, 1 - \sin x\cos x = \frac{4}{3}$$

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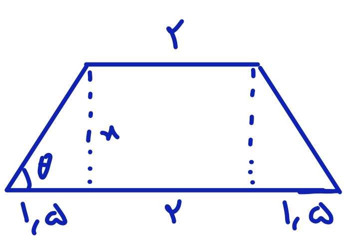
۴)



$$\cos\theta = 0.6 \longrightarrow \cot\theta = \frac{3}{4}$$

$$\frac{1.5}{x} = \frac{3}{4} \longrightarrow x = 2$$

$$S = \frac{(2+5)\times 2}{2} = 7$$

$$\cos\theta = \frac{3}{5} = \frac{6}{10} \quad (x \to 3)$$

$$\sin\theta = \frac{h}{5} = \frac{8}{10} \quad (h \to 4)$$

$$S = \frac{(2+8)}{2} \times 4 = 20$$

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۵)

$$\tan(285°)\tan(-145°) - \sin(195°)\cos(255°)$$

$$\tan\left(\frac{3\pi}{2}+\alpha\right)\tan(\pi+\alpha) - \sin\alpha\cos\left(\frac{3\pi}{2}-\alpha\right) = \underbrace{-\cot\alpha\tan\alpha}_{-1} + \sin^2\alpha = -\cos^2\alpha \Rightarrow k = -1$$

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۶)

$$\sqrt{3} \times \left(-\frac{\sqrt{3}}{2}\right) \times (-\cos\alpha) - \sqrt{2} \times \left(\frac{\sqrt{2}}{2}\right) \times (\cos\alpha) = 1.5\cos\alpha - \cos\alpha = 0.5\cos\alpha$$

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۷)

$$f(x) = 16\cos^2(3x)\underbrace{\cos^2(6x)}_{\frac{3}{4}}\underbrace{\cos^2(12x)\cos^2(24x)}_{\frac{1}{16}} = \frac{3}{4}\cos^2\frac{\pi}{12} \longrightarrow \frac{6+3\sqrt{3}}{16}$$

$$\cos^2\frac{\pi}{12} = \frac{1+\frac{\sqrt{3}}{2}}{2} = \frac{2+\sqrt{3}}{4}$$

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۸)

$$1 - \sin x = 4 + 4\sin x \longrightarrow \sin x = -\frac{3}{5} \,,\, \cos x = -\frac{4}{5} \longrightarrow \tan\frac{x}{2} = \frac{-\frac{3}{5}}{1-\frac{4}{5}} = -3$$

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۹)

$$\frac{\sin\theta}{1+\cos\theta} + \frac{1-\cos\theta}{\sin\theta} = \cot\frac{\theta}{2} + \cot\frac{\theta}{2} = 2\cot\frac{\theta}{2} \longrightarrow k = 2$$

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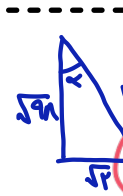
۱۰)

$$\cos\left(\frac{3\pi}{4}+\alpha\right) = \cos\frac{3\pi}{4}\cos\alpha - \sin\frac{3\pi}{4}\sin\alpha = \left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{7\sqrt{2}}{10}\right) - \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{10}\right) = \frac{7}{10} + \frac{1}{10} = \frac{4}{5}$$

$$\cos\alpha = -\frac{7\sqrt{2}}{10} \qquad \sin\alpha = \frac{\sqrt{2}}{10}$$



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۹) $A = \sqrt{۳}\times\frac{-\sqrt{۳}}{۲}\times\sin(۲۷۰-۲۷) - \sqrt{۲}\times\frac{\sqrt{۲}}{۲}\cos(۱۸۰-۲۷)$

$\rightarrow \frac{۵}{۲}\cos(۲۷)$ $\rightarrow$ برابر $\frac{۵}{۲}$

۱۰) $\cos\left(\frac{۱۱\pi}{۴}+\alpha\right) = -\left(\cos\alpha\cos\frac{\pi}{۴}+\sin\alpha\sin\frac{\pi}{۴}\right)$

$\rightarrow \frac{-\sqrt{۲}}{۲}(\cos\alpha+\sin\alpha)$ $\quad \cos\alpha = \frac{-۷\sqrt{۲}}{۱۰}$

$\hookrightarrow -\frac{\sqrt{۲}}{۲}\left(\frac{-۷\sqrt{۲}}{۱۰}+\frac{\sqrt{۲}}{۱۰}\right) = \frac{۳}{۵}$