یازدهم دفتر C — فاطمه ... - ۲۸ — ۲۰

Date No

1- $\cot\alpha = \dfrac{\cos\alpha}{\sqrt{1-\cos^۲\alpha}} = \dfrac{\cos\alpha}{\sqrt{\sin^۲\alpha}} \Rightarrow \dfrac{\cos\alpha}{\sin\alpha} = \dfrac{\cos\alpha}{|\sin\alpha|} \Rightarrow \sin\alpha > ۰$ ①

$\dfrac{1}{\sqrt{\cos^۲\alpha}} - \dfrac{1}{\cot\alpha} = \dfrac{1-\sin\alpha}{|\cos\alpha|} \longrightarrow \dfrac{1}{|\cos\alpha|} - \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{1-\sin\alpha}{|\cos\alpha|} \longrightarrow \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{\sin\alpha}{|\cos\alpha|} \Rightarrow \cos\alpha > ۰$ ②

①,② $\longrightarrow$ α در ربع اول

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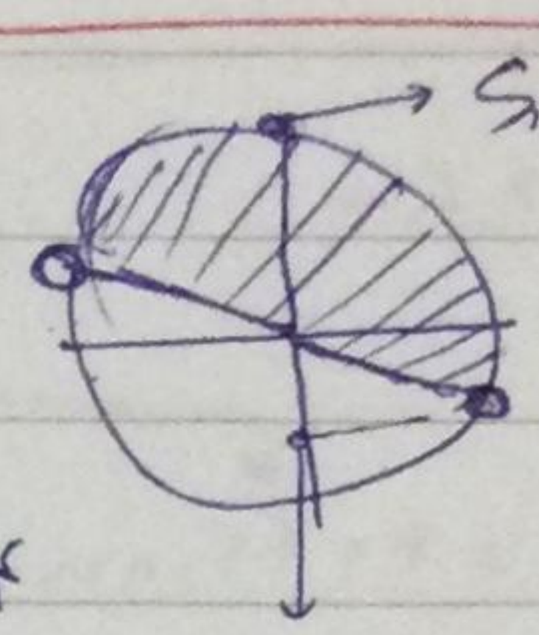
2- $\sin ۲\alpha = \dfrac{m-1}{۴}$ $\quad -\dfrac{\pi}{۱۲} < x < \dfrac{۵\pi}{۲۴} \Rightarrow -\dfrac{\pi}{۶} < ۲x < \dfrac{۵\pi}{۶}$



$\longrightarrow -\dfrac{1}{۲} < \sin ۲x \leq 1 \Rightarrow -\dfrac{1}{۲} < \dfrac{m-1}{۴} \leq 1 \Rightarrow -۲ < m-1 \leq ۴$

$\Rightarrow -1 < m \leq ۵$

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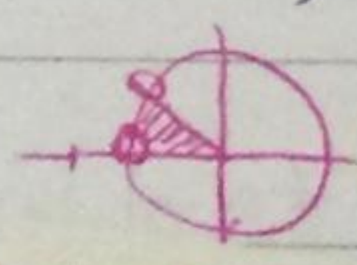
3- $\tan x + \cot x = -۳$ $\quad ۳\pi < ۴x < ۴\pi$ $\quad A = \dfrac{1}{\sin^۳x + \cos^۳x} = \dfrac{1}{(\sin x+\cos x)(1-\sin x\cos x)}$

$\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x} = -۳ \Rightarrow \dfrac{1}{\sin x\cos x} = -۳ \Rightarrow \sin x\cos x = -\dfrac{1}{۳}$

$\dfrac{۳\pi}{۴} < x < \pi$



$|\cos x| > |\sin x| \Rightarrow \cos x + \sin x < ۰$

$(\sin x+\cos x)^۲ = \sin^۲x + \cos^۲x + ۲\sin x\cos x = 1 - \dfrac{۲}{۳} \Rightarrow |\sin x+\cos x| = \dfrac{\sqrt{۳}}{۳} \longrightarrow \sin x + \cos x = -\dfrac{\sqrt{۳}}{۳}$

$\Rightarrow A = \dfrac{1}{-\dfrac{\sqrt{۳}}{۳} \times \dfrac{۴}{۳}} = -\dfrac{۹}{۴\sqrt{۳}}$

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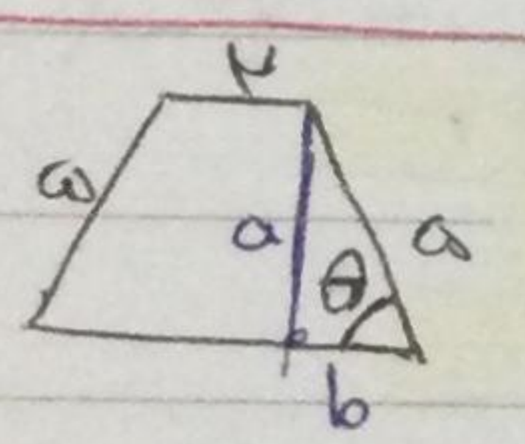

$\longrightarrow \cos\theta = ۰{,}۶ = \dfrac{b}{۵} = \dfrac{۶}{۱۰} \Rightarrow b = ۳$

$\Downarrow$

$\sin\theta = ۰{,}۸ = \dfrac{a}{۵} = \dfrac{۸}{۱۰} \Rightarrow a = ۴$

$S_{\text{ذوزنقه}} = \dfrac{(۲+۲+۲b)\times a}{۲} = \dfrac{۱۰\times۴}{۲} = ۲۰$

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5- $\tan(۲۸۵)\tan(-۱۶۵) - \sin(۱۰۹۵)\cos(۲۵۵) = -\cot ۱۵ \times \tan ۱۵ - \sin ۱۵ \cdot (-\sin ۱۵)$

($۲۸۵ = \frac{۳\pi}{۲}+۱۵$, $-۱۶۵ = -(\pi-۱۵)$, $۱۰۹۵ = ۶\pi+۱۵$, $۲۵۵ = \frac{۳\pi}{۲}-۱۵$)

$\Rightarrow -1 + \sin^۲ ۱۵ = -1 + 1 - \cos^۲ ۱۵ = k\cos^۲ ۱۵ \Rightarrow k = -1$

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6- $A = \sqrt{۳}\cos(۲۱۰)\sin(۲۴۳) - \sqrt{۲}\sin(۱۳۵)\cos(۱۵۳)$

$A = \sqrt{۳} \times \dfrac{\sqrt{۳}}{-۲} \times (-\cos ۲۷) - \sqrt{۲} \times \dfrac{\sqrt{۲}}{-۲} \times \cos ۲۷ = \dfrac{۳}{۲}\times\cos ۲۷ + \cos ۲۷ = \dfrac{۵}{۲}\cos ۲۷$

$\Rightarrow \dfrac{A}{\cos ۲۷} = \dfrac{۵}{۲}$

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dotnote

7- $f(x) = 16\cos^2(3x)\cos^2(6x)\cos^2(12x)\cos^2(24x)$ $\qquad 2\cos^2\alpha = 1+\cos 2\alpha$

$\longrightarrow f(x) = (1+\cos(6x))(1+\cos(12x))(1+\cos(24x))(1+\cos(48x))$

$\Rightarrow f(\frac{\pi}{36}) = 8(1+\cos\frac{\pi}{6})(\cos^2\frac{\pi}{6})(\cos^2\frac{\pi}{3})(\cos^2\frac{2\pi}{3}) = 8(1+\frac{\sqrt{3}}{2})(\frac{3}{4})(\frac{1}{4})(\frac{1}{4}) = \frac{3\sqrt{3}+6}{16}$

$\frac{\sqrt{3}+2}{2}$

8- $\sin x < 0 , \cos x < 0$ $\qquad \frac{1-\sin x}{1+\sin x} = \frac{4}{1}$ $\qquad \tan\frac{x}{2}$

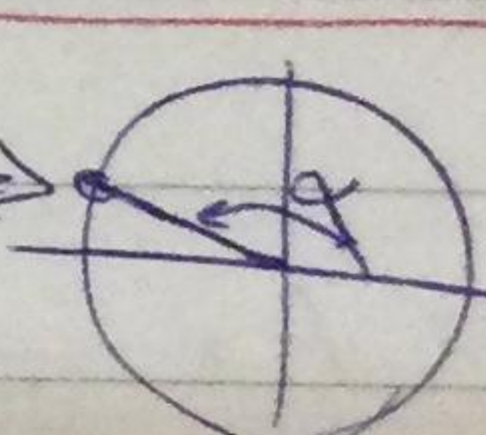
$1-\sin x = 4+4\sin x \Rightarrow -\frac{3}{5} = \sin x \Rightarrow \cos x = -\sqrt{1-\frac{9}{25}} = -\frac{4}{5}$

$\tan\frac{x}{2} = \frac{\sin\alpha}{1+\cos\alpha} = \frac{-\frac{3}{5}}{\frac{1}{5}} = -3$

9- $\frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = k\cot\frac{\theta}{2}$

$\cot\frac{\theta}{2} + \cot\frac{\theta}{2} = k\cot\frac{\theta}{2} \Rightarrow k = 2$

10- $\sin\alpha = \frac{\sqrt{2}}{10}$ $\qquad \sin\alpha > 0 \quad \cos\alpha < 0$ $\Rightarrow \cos\alpha = -\sqrt{1-\frac{2}{100}} = -\frac{\sqrt{98}}{10} \Rightarrow$



$\cos(\frac{11\pi}{4}+\alpha) = \cos(\frac{3\pi}{4}+\alpha) = \cos\frac{3\pi}{4}\times\cos\alpha - \sin\frac{3\pi}{4}\times\sin\alpha = -\frac{\sqrt{2}}{2}\times -\frac{\sqrt{98}}{10} - \frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{10}$

$2\pi + \frac{3\pi}{4}$

$\longrightarrow \frac{14}{20} - \frac{2}{20} = \frac{12}{20} = \frac{3}{5}$