ترانه طاهریان - یازدهم دختر - تکلیف ۲۸

-۱

$$\cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} = \frac{\cos\alpha}{\sqrt{\sin^2\alpha}} = \frac{\cos\alpha}{|\sin\alpha|}$$

$$\Rightarrow |\sin\alpha| = \sin\alpha \Rightarrow \sin\alpha > 0$$

$$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|}$$

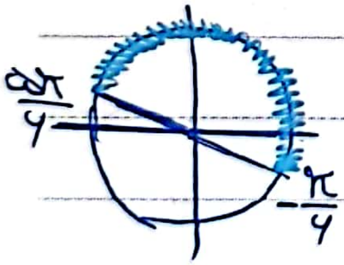
انتهای کمان α در ناحیه‌ی اول مثلثاتی است

$$\Rightarrow \cancel{\frac{1}{|\cos\alpha|}} - \frac{\sin\alpha}{\cos\alpha} = \cancel{\frac{1}{|\cos\alpha|}} - \frac{\sin\alpha}{|\cos\alpha|}$$

$$\Rightarrow |\cos\alpha| = \cos\alpha \Rightarrow \cos\alpha > 0$$

-۲

$$-\frac{\pi}{12} < x < \frac{5\pi}{12} \xrightarrow{\times 2} -\frac{\pi}{6} < 2x < \frac{5\pi}{6}$$



$$\rightsquigarrow -\frac{1}{2} < \sin 2x \leqslant 1 \Rightarrow -\frac{1}{2} < \frac{m-1}{4} \leqslant 1$$

$$\xrightarrow{\times 4} -2 < m-1 \leqslant 4 \xrightarrow{+1} -1 < m \leqslant 5$$

$$\Rightarrow \boxed{m \in (-1, 5]}$$

۳-

$$\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}=\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}=$$

$$\frac{1}{\sin x\cos x}=-3 \Rightarrow \sin x\cos x=\frac{-1}{3}$$

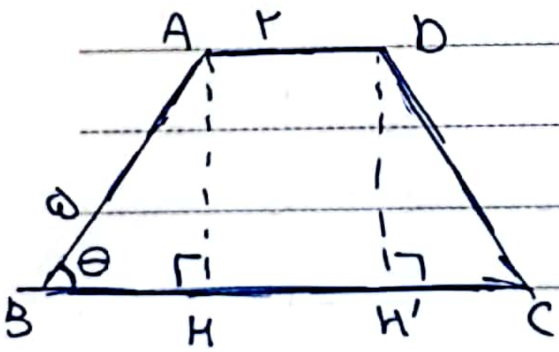
$$(\sin x+\cos x)^2=\sin^2 x+\cos^2 x+2\sin x\cos x$$

$$=1+2\times\left(\frac{-1}{3}\right)=1-\frac{2}{3}=\frac{1}{3} \Rightarrow \sin x+\cos x=\frac{1}{\sqrt{3}}$$

$$\frac{1}{\sin^3 x+\cos^3 x}=\frac{1}{(\sin x+\cos x)}\times\frac{1}{(\sin^2 x+\cos^2 x-\sin x\cos x)}$$

$$=\sqrt{3}\times\frac{1}{\underbrace{1-\left(\frac{-1}{3}\right)}_{\frac{4}{3}}}=\sqrt{3}\times\frac{3}{4}=\boxed{\frac{3\sqrt{3}}{4}}$$

۴- ارتفاع ذوزنقه را رسم می‌کنیم، در مثلث $\overset{\triangle}{ABH}$:



$$\cos\theta=\frac{BH}{AB}=\frac{BH}{5}=0/6$$

$$\Rightarrow BH=3 \quad , AH=4$$

$$S_{ABCD}=\underbrace{S_{ABH}+S_{DCH'}}_{2\times S_{ABH}=2\times\frac{3\times 4}{2}=12}+S_{ADHH'}=12+8=20$$

$$S_{ADHH'} \rightarrow 2\times 4=8$$

۵- $\tan\left(\frac{۳\pi}{۲}+۱۵^\circ\right)\tan(۱۵^\circ-\pi)-\sin(۱۵^\circ)\cos\left(\frac{۳\pi}{۲}-۱۵\right)$

$= (-\cot ۱۵^\circ)\times \tan ۱۵^\circ - \sin ۱۵^\circ \times (-\sin ۱۵^\circ)$

$= -۱ + \sin^۲ ۱۵ = -\cos^۲ ۱۵ \Rightarrow \boxed{k=-۱}$

۶- $\sqrt{۳}\cos(۲۱۰^\circ)\times\sin\left(\frac{۳\pi}{۲}-۲۷^\circ\right)-\sqrt{۲}\sin(۱۳۵^\circ)\times\cos(\pi-۲۷^\circ)$

$= \sqrt{۳}\times\frac{-\sqrt{۳}}{۲}\times(-\cos ۲۷^\circ) - \sqrt{۲}\times\frac{\sqrt{۲}}{۲}\times(-\cos ۲۷^\circ)$

$\frac{۳}{۲}\cos ۲۷^\circ + \cos ۲۷^\circ = \frac{۵}{۲}\cos ۲۷^\circ$

$\frac{۵}{۲}$ برابر

۷- $f\left(\frac{\pi}{۳۶}\right) = ۱۶\cos^۲\left(\frac{\pi}{۱۲}\right)\times\cos^۲\left(\frac{\pi}{۶}\right)\times\cos^۲\left(\frac{\pi}{۳}\right)$

$\times\cos^۲\left(\frac{۲\pi}{۳}\right) = ۱۶\times\frac{۳}{۴}\times\frac{۱}{۴}\times\frac{۱}{۴}\times\cos^۲\left(\frac{\pi}{۱۲}\right)$

$= \frac{۳}{۴}\cos^۲\left(\frac{\pi}{۱۲}\right) = \frac{۳}{۴}\times\frac{\sqrt{۳}+۲}{۴} = \boxed{\frac{۳\sqrt{۳}+۶}{۱۶}}$

$\cos ۲\alpha = \cos^۲\alpha - \sin^۲\alpha \Rightarrow \cos\frac{\pi}{۶} = \cos^۲\frac{\pi}{۱۲} - \sin^۲\frac{\pi}{۱۲}$

$= \frac{\sqrt{۳}}{۲} \Rightarrow \begin{cases} \cos^۲\frac{\pi}{۱۲} - \sin^۲\frac{\pi}{۱۲} = \frac{\sqrt{۳}}{۲} \\ \cos^۲\frac{\pi}{۱۲} + \sin^۲\frac{\pi}{۱۲} = ۱ \end{cases}$

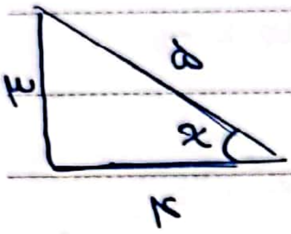
$۲\cos^۲\frac{\pi}{۱۲} = ۱+\frac{\sqrt{۳}+۲}{۲} \Rightarrow \cos^۲\frac{\pi}{۱۲} = \frac{\sqrt{۳}+۲}{۴}$

۸-

$$\frac{1-\sin x}{1+\sin x}=4 \Rightarrow 4+4\sin x = 1-\sin x$$

$$\Rightarrow 5\sin x = -3$$

$$\Rightarrow \sin x = \frac{-3}{5}$$



$$\Rightarrow \cos x = \frac{-4}{5}$$

$$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x} = \frac{\frac{-3}{5}}{1-\frac{4}{5}} = \frac{\frac{-3}{5}}{\frac{1}{5}} = \boxed{-3}$$

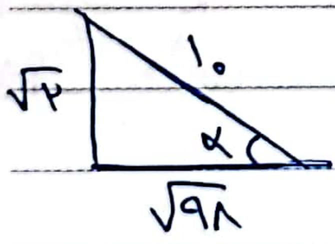
۹-

$$\frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha} = \tan\frac{\alpha}{2}$$

$$\frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = \cot\frac{\theta}{2} + \cot\frac{\theta}{2}$$

$$= 2\cot\frac{\theta}{2} \Rightarrow k = \boxed{2}$$

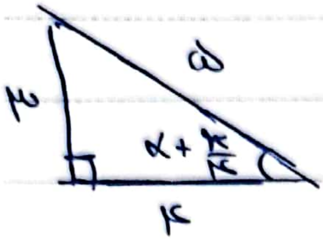
۱۰-



$$\sin\alpha = \frac{\sqrt{2}}{10} \quad \cos\alpha = \frac{-7\sqrt{2}}{10}$$

$$\sqrt{2}\sin\left(\alpha+\frac{\pi}{4}\right) = \sin\alpha + \cos\alpha = \frac{\sqrt{2}-7\sqrt{2}}{10} = \frac{-6\sqrt{2}}{10}$$

$$\Rightarrow \sin\left(\alpha+\frac{\pi}{4}\right) = -\frac{3}{5}$$



$$\Rightarrow \cos\left(\alpha+\frac{\pi}{4}\right) = -\frac{4}{5}$$

$$|\sin\alpha| < |\cos\alpha| \Rightarrow \frac{3\pi}{4} < \alpha < \pi$$

$$\Rightarrow \pi < \alpha+\frac{\pi}{4} < \frac{5\pi}{4} \Rightarrow \cos\left(\alpha+\frac{\pi}{4}\right) < 0$$

$$\cos\left(\cancel{\frac{11\pi}{4}}+\alpha\right) = \cos\left(\pi-\left(\frac{\pi}{4}+\alpha\right)\right) = -\cos\left(\alpha+\frac{\pi}{4}\right)$$

$$\frac{3\pi}{4}$$

$$= \boxed{+\frac{4}{5}}$$