نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

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$\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} = \frac{\cos\alpha}{|\sin\alpha|}$ $\quad \frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} = \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha}$

$= \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{|\cos\alpha|} \Rightarrow \cos\alpha = |\cos\alpha| \rightarrow \cos\alpha > 0$ ناحیه‌ی اول یا چهارم

$\cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow |\sin\alpha| = \sin\alpha \Rightarrow \sin\alpha > 0$ ناحیه‌ی اول یا دوم

$\Rightarrow$ ناحیه‌ی اول ← جواب

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$-\frac{\pi}{12} < u < \frac{3\pi}{12} \Rightarrow -\frac{2\pi}{12} < 2u < \frac{10\pi}{12} \Rightarrow -\frac{\pi}{6} < 2u < \frac{3\pi}{4}$

$-\frac{1}{2} < \sin 2\alpha \leq 1 \Rightarrow -\frac{1}{2} < \frac{m-1}{2} \leq 1 \Rightarrow -2 < m-1 \leq 2 \Rightarrow -1 < m \leq 3$ ← جواب

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$\tan u + \cot u = -2 \Rightarrow \frac{1}{\sin u \cos u} = -2 \Rightarrow \sin u \cos u = -\frac{1}{2}$

$\frac{\pi}{2} < u < \pi \Rightarrow \frac{3\pi}{4} < u < \pi$

$\frac{1}{\sin^3 u + \cos^3 u} = \frac{1}{(\sin u + \cos u)(\sin^2 u + \cos^2 u - \sin u \cos u)} = -\frac{1}{\frac{3\sqrt{3}}{4}} = -\frac{4}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = -\frac{4\sqrt{3}}{9}$

$(\sin u + \cos u)^2 = \sin^2 u + \cos^2 u + 2\sin u \cos u \Rightarrow \sin u + \cos u = \pm\frac{\sqrt{6}}{2}$ ← ناحیه‌ی دوم

$= -\frac{\sqrt{6}}{2}$

$\frac{1}{\sin^3 u + \cos^3 u} = -\frac{2\sqrt{6}}{3}$ ← جواب

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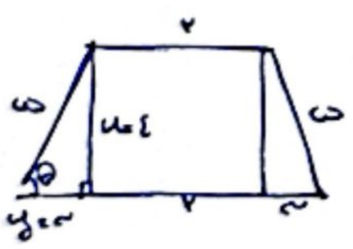

$S = \frac{1}{2}(2+8) \times 6 = 30$ ← جواب

$\cos\theta = \frac{4}{5} = 0.8 \Rightarrow y = 3u = 6$

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$10^\circ = \alpha$

$280^\circ = 270^\circ + 10^\circ$

$-140^\circ = -180^\circ + 10^\circ$

$690^\circ = 720^\circ + 10^\circ$

$-260^\circ = 270^\circ - 10^\circ$

$\tan\left(\frac{3\pi}{2}+\alpha\right)\tan(-\pi+\alpha) - \sin(4\pi+\alpha)\cos\left(\frac{3\pi}{2}-\alpha\right)$

$= \underbrace{(-\cot\alpha)(\tan\alpha)}_{-1} - \underbrace{(\sin\alpha)(-\sin\alpha)}_{-\sin^2\alpha} = -1 - (-\sin^2\alpha) = -1 + \sin^2\alpha = -\cos^2\alpha$

$K = -1$ ← جواب

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$\alpha = 240^\circ$

$\sqrt{2}\underbrace{\cos(210^\circ)}_{-\frac{\sqrt{3}}{2}}\sin(240^\circ) - \sqrt{2}\underbrace{\sin(180^\circ)}_{\frac{\sqrt{3}}{2}}\cos(180^\circ) = -\frac{\sqrt{2}}{2}\sin\left(\frac{3\pi}{2}-\alpha\right) - \cos(\pi-\alpha) =$

$\underbrace{\qquad\qquad}_{-\frac{\sqrt{6}}{2}}$ $\underbrace{\qquad}_{-1}$

$-\frac{\sqrt{2}}{2}(-\cos\alpha) + \cos\alpha = \frac{\sqrt{2}}{2}\cos\alpha + \cos\alpha = \frac{\sqrt{2}+2}{2}\cos\alpha \Rightarrow$ مخرج $\frac{\sqrt{2}+2}{2}$ کم جواب

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$\cos\frac{\pi}{8} = \cos 15^\circ = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ = \frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{2} + \frac{1}{2}\times\frac{\sqrt{2}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4} = \sqrt{\frac{2+\sqrt{3}}{4}}$

$f\left(\frac{\pi}{24}\right) = 14\cos^2\left(\frac{3\pi}{24}\right)\cos^2\left(\frac{4\pi}{24}\right)\cos^2\left(\frac{12\pi}{24}\right)\cos^2\left(\frac{21\pi}{24}\right) = 14\underbrace{\cos^2\left(\frac{\pi}{8}\right)}_{\frac{2+\sqrt{3}}{4}}\underbrace{\cos^2\left(\frac{\pi}{6}\right)}_{\frac{3}{4}}\underbrace{\cos^2\left(\frac{\pi}{2}\right)}_{0}\underbrace{\cos^2\left(\frac{7\pi}{8}\right)}_{\frac{1}{2}}$

$14\left(\frac{2+\sqrt{3}}{4}\right)\times\frac{3}{4}\times 0\times\frac{1}{2} = \frac{3}{2}\times\frac{3+2\sqrt{3}}{16} = \frac{9+6\sqrt{3}}{16}$ کم جواب

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$1-\sin\alpha = 3+3\sin\alpha \Rightarrow 2\sin\alpha = -2 \Rightarrow \sin\theta = -\frac{2}{3}$

$\cos\theta = -\frac{\sqrt{5}}{3}$

$\tan\frac{\alpha}{2} = \frac{\sin\theta}{1+\cos\theta} = \frac{-\frac{2}{3}}{\frac{1}{3}} = -2$ جواب

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$\frac{1-\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta} = \tan\frac{\theta}{2}$ چون $\Rightarrow \frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = \cot\frac{\theta}{2} + \cot\frac{\theta}{2} = 2\cot\frac{\theta}{2} = k$ کم جواب $k=2$

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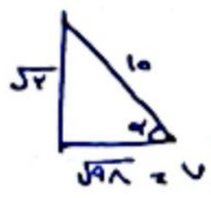

$\cos\theta = -\frac{7\sqrt{2}}{10}$ نسبت‌ها

$\cos\left(\frac{11\pi}{4}+\alpha\right) = \underbrace{\cos\frac{11\pi}{4}}_{\cos(2\pi+\frac{3\pi}{4}) = -\frac{\sqrt{2}}{2}}\cos\alpha - \sin\frac{11\pi}{4}\sin\alpha = -\frac{\sqrt{2}}{2}\times\frac{-7\sqrt{2}}{10} = \frac{\sqrt{2}}{2}\times\frac{7\sqrt{2}}{10}$

$= \frac{7}{10} - \frac{1}{10} = 0/6$ کم جواب