(۱) $\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} \quad \frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot} = \frac{1-\sin\alpha}{|\cos\alpha|}$

$\frac{\cos\alpha}{\sin\alpha} = \frac{\cos}{\sqrt{\sin^2\alpha}} \rightarrow \sqrt{\sin^2} = \sin \Rightarrow \sin > 0$

$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\frac{\cos\alpha}{\sin\alpha}} = \frac{1-\sin\alpha}{|\cos\alpha|}$

$\hookrightarrow \frac{1}{\sqrt{\cos^2\alpha}} - \frac{\sin\alpha}{\cos\alpha} \Rightarrow \sqrt{\cos^2\alpha} = \cos\alpha \Rightarrow \cos\alpha > 0$

$\Rightarrow$ جواب اول

(۲) $-\frac{\pi}{4} < 2x < \frac{3\pi}{4} \rightarrow -\frac{1}{2} < \sin 2x \leq 1 \rightarrow -\frac{1}{2} < \frac{m-1}{3} \leq 1 \rightarrow -3 < m \leq 5$

(۳) $\frac{1}{\sin x \cos x} = -3 \rightarrow \sin x \cdot \cos x = -\frac{1}{3}$

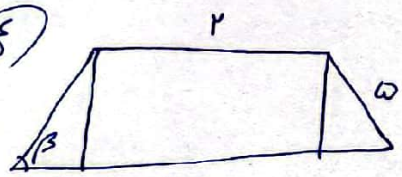
$\cos^3 x + \sin^3 x = (\cos x + \sin x)(\sin^2 x + \cos^2 x - \sin x \cos x) = -\frac{3\sqrt{3}}{4}$

$A^2 = 1 + 2\sin x \cos x = 1 + 2\left(-\frac{1}{3}\right) = \frac{1}{3} \rightarrow A = \pm\frac{1}{\sqrt{3}} \xrightarrow{A<0} A = -\frac{1}{\sqrt{3}}$

$\frac{1}{\cos^3 x + \sin^3 x} = -\frac{4}{3\sqrt{3}}$

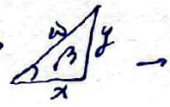
جواب گزینه ۳

(۴)



$\rightarrow$ محیط $= 2 + 3 + 3 + 8$

$\rightarrow$ مساحت $= \frac{(3+8)\times 4}{2} = 22$



$\rightarrow \cos\beta = \frac{x}{\omega} = \frac{1}{2} \rightarrow x = 3$

$\gamma = 4$

(۵) $\tan(2\omega\pi) = \tan(2\omega\pi - 2\pi) = \tan(-7\omega) = -\tan(7\omega)$

$\tan(-14\omega) = -\tan(14\omega) = -(-\tan(1\omega)) = \tan(1\omega)$

$\sin(1.9\omega) = \sin(10\omega - 4\pi) = \sin(12\omega) = \sin(3\cdot 4\omega - 2\pi) = \sin(1\omega)$

$\cos(2\omega\pi) = \cos(2\omega\pi - 2\pi) = \cos(-10\omega) = -\sin(1\omega)$

$-\tan(7\omega)\tan(1\omega) - (-\sin(1\omega)\sin(1\omega)) = -1 + \sin^2 1\omega = -1 + 1 - \cos^2(1\omega) = -\cos^2(1\omega)$

$= \delta_{k=1}$

۶ $A = -\frac{۳}{۴} \times -\cos ۲V + \frac{۲}{۴}\cos ۲V = \frac{۳}{۴}\cos ۲V \longrightarrow$ بزرگترین $\frac{۳}{۴}$

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۷ $f(x) = ۱۴\cos^۲(۳x)\cos^۲(۶x)\cos^۲(۱۲x)\cos^۲(۲۴x)$

$f(\frac{\pi}{۲۴}) = ۱۴\cos^۲(\frac{\pi}{۱۲})\cos^۲(\frac{\pi}{۶})\cos^۲(\frac{\pi}{۳})\cos^۲(\frac{۳\pi}{۴})$

$= ۱۴\cos^۲(۱۵)\cos^۲(۳۰)\cos^۲(۶۰)\cos^۲(۱۳۵)$

$= \frac{۳}{۴}\cos^۲(۱۵) \longrightarrow \frac{۳}{۴}(\frac{۱+\cos ۳۰}{۲}) = \frac{۶+۳\sqrt{۳}}{۱۶}$

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۸ $۱-\sin x = ۴+۴\sin x \rightarrow ۵\sin x = -۳ \longrightarrow \sin = -\frac{۳}{۵}$

$\sin^۲+\cos^۲=۱ \longrightarrow \frac{۹}{۲۵}+\cos^۲=۱ \rightarrow \cos^۲=\frac{۱۶}{۲۵} \longrightarrow \cos = -\frac{۴}{۵}$

$\tan x = \frac{-\frac{۳}{۵}}{-\frac{۴}{۵}} = \frac{۳}{۴}$

$\tan x = \frac{۲\tan\frac{x}{۲}}{۱-\tan^۲\frac{x}{۲}} \xrightarrow{\tan\frac{x}{۲}=z} \frac{۳}{۴} = \frac{۲z}{۱-z^۲} \rightarrow \begin{cases} z = -۳ \times \\ z = \frac{۱}{۳} \checkmark \end{cases}$

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۹ $\frac{۲\sin\frac{\theta}{۲}\cos\frac{\theta}{۳}}{۲\sin^۲\frac{\theta}{۲}} + \frac{۲\cos^۲\frac{\theta}{۳}}{۲\sin\frac{\theta}{۲}\cos\frac{\theta}{۳}} = \cot\frac{\theta}{۲} + \cot\frac{\theta}{۲} = ۲\cot\frac{\theta}{۲}$

$\rightarrow k = ۲$

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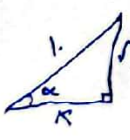
۱۰ $\cos(\frac{۱۱\pi}{۴}+\alpha) = \cos(۳\pi - \frac{\pi}{۴}+\alpha) = -\cos(\alpha - \frac{\pi}{۴}) = -(\cos\alpha\cos\frac{\pi}{۴}+\sin\alpha\sin\frac{\pi}{۴})$

$= -\frac{\sqrt{۲}}{۲}(\cos\alpha+\sin\alpha) \Rightarrow \cos(\frac{۱۱\pi}{۴}+\alpha) = -\frac{\sqrt{۲}}{۲}(-\frac{۷\sqrt{۲}}{۱۰}+\frac{\sqrt{۲}}{۱۰}) = \frac{۳}{۵}$



$\rightarrow k = \sqrt{۹۸} \longrightarrow \cos\alpha = -\frac{۷\sqrt{۲}}{۱۰}$