تکلیف (27)

فاطمه مختاری

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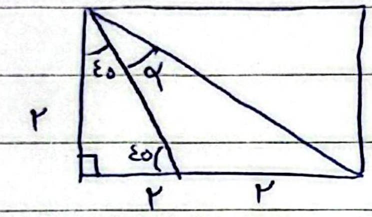
① $S=\frac{1}{2}ab\sin\alpha \Rightarrow \frac{1}{2}\times\sqrt{3}\times 6\times\sin\alpha=4{,}5$

$\sin\alpha=\frac{4{,}5}{3\sqrt{3}}=\frac{\sqrt{3}}{2}\rightarrow \alpha=\begin{matrix}60^\circ\\120^\circ\end{matrix}$ $\boxed{\frac{120^\circ}{60^\circ}\Rightarrow 2 \text{ برابر}}$

②



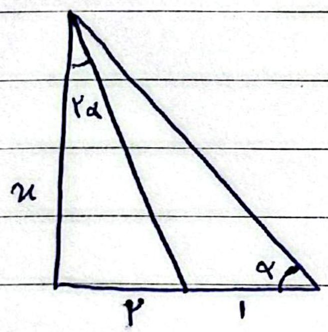

$\cot(\alpha+45)=\frac{2}{4}=\frac{1}{2}$

$\cot(\alpha+45)=\frac{1-\tan\alpha\tan45^\circ}{\tan\alpha+\tan45}=\frac{1-\tan\alpha}{\tan\alpha+1}$

$\frac{1-\tan\alpha}{\tan\alpha+1}=\frac{1}{2}\rightarrow 2-2\tan\alpha=1+\tan\alpha$

$3\tan\alpha=1\rightarrow \boxed{\tan\alpha=\frac{1}{3}\quad \cot\alpha=3}$

③




$\tan\alpha=\frac{x}{3}$

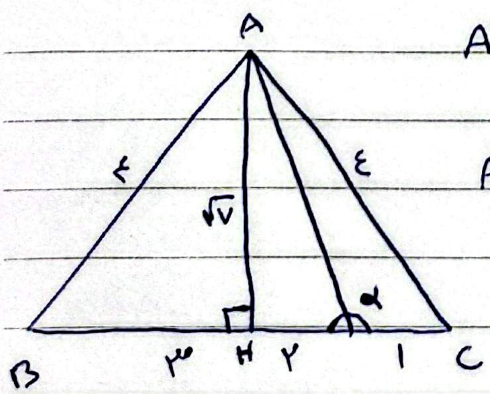
$\tan2\alpha=\frac{2}{x}\Rightarrow \tan2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}=\frac{\frac{2x}{3}}{\frac{9-x^2}{9}}=\frac{6x}{9-x^2}=\frac{2}{x}$

$6x^2=18-2x^2\qquad 8x^2=18$

$x^2=\frac{9}{4}\rightarrow x=\pm\frac{3}{2}$ غ ق ق

$\tan\alpha=\frac{\frac{3}{2}}{3}\Rightarrow\frac{1}{2}\rightarrow\boxed{\cot\alpha=2}$

④




$AH\rightarrow$ ارتفاع و میانه

$\overset{\Delta}{AHC}=AH^2+9=16$

$AH=\sqrt{7}$

$\overset{\Delta}{AEH}\rightarrow \tan(180-\alpha)=\frac{\sqrt{7}}{2}$

$\tan(180-\alpha)=-\tan\alpha\rightarrow \boxed{\tan\alpha=-\frac{\sqrt{7}}{2}}$

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(۵)

$$\underbrace{\sin^2 x + \sin^2 x + \cos^2 x}_{1} = \frac{4}{3} \to \sin^2 x = \frac{1}{3}$$

$$\sin x = \pm\frac{1}{\sqrt{3}} \qquad \cos x = \pm\frac{\sqrt{2}}{\sqrt{3}}$$

$1,\ \sqrt{2},\ \sqrt{3}$

$$\tan x = \frac{1}{\sqrt{2}}$$

$$\boxed{\tan^2 x = \frac{1}{2}}$$

(۶)

$$\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha} = \frac{\sin^4\alpha + 4(1-\sin^2\alpha)}{1+(1-\sin^2\alpha)} - \frac{\cos^4\alpha + 4(1-\cos^2\alpha)}{1+(1-\cos^2\alpha)}$$

$$\frac{\sin^4\alpha + 4 - 4\sin^2\alpha}{2-\sin^2\alpha} - \frac{\cos^4\alpha + 4 - 4\cos^2\alpha}{2-\cos^2\alpha}$$

$$(2-\sin^2\alpha)\quad \frac{(2-\sin^2\alpha)^2}{\cancel{2-\sin^2\alpha}} - \frac{(2-\cos^2\alpha)^2}{\cancel{2-\cos^2\alpha}} \Rightarrow (2-\sin^2\alpha) - 2 + \cos^2\alpha = \cos^2\alpha - \sin^2\alpha = \cos 2\alpha$$

(۷)

($\tan\alpha = \frac{4}{3}$ ، $\alpha$ در ربع سوم)

$$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right) - \tan\left(\alpha - \frac{3\pi}{2}\right)$$

$3,\ 4,\ 5$

Sin ، Cos منفی

tan ، Cot ⇐ مثبت

$$(\cos\alpha)\times(-\sin\alpha) + \cot\alpha$$

$$-\frac{3}{5}\times\frac{4}{5} + \frac{3}{4} \Rightarrow -\frac{12}{25} + \frac{3}{4} = \boxed{\frac{27}{100}}$$

(۸)

$$3\cos 4x + \sqrt{2}\sin x - \sqrt{2}\cos x \qquad x = \frac{\pi}{12}$$

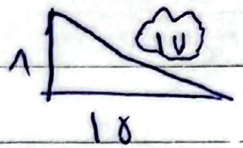
$$3\cos\frac{\pi}{3} + \sqrt{2}(\sin x - \cos x)$$

$$3\cos\frac{\pi}{3} + \sqrt{2}\left(\sqrt{2}\sin\left(\frac{\pi}{12}-\frac{\pi}{4}\right)\right) \to \frac{3}{2} + \sqrt{2}\left(\sqrt{2}\times\sin\left(-\frac{\pi}{6}\right)\right) = \frac{3}{2} + \sqrt{2}\left(\sqrt{2}\times -\frac{1}{2}\right)$$

$$\frac{3}{2} - 1 = \boxed{\frac{1}{2}}$$

(9)

$$\tan\alpha = \frac{2\tan(\frac{\alpha}{2})}{1-\tan^2(\frac{\alpha}{2})} = \frac{2\times\frac{1}{4}}{1-\frac{1}{16}} = \frac{\frac{1}{2}}{\frac{15}{16}} \Rightarrow \boxed{\frac{16}{30} = \frac{8}{15}}$$




$$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{\frac{136-120}{255}}{\frac{136-255}{289}} \Rightarrow \frac{\frac{16}{255}}{\frac{-119}{289}} = \boxed{-\frac{16}{105}}$$

(10)

$$0 < \frac{\cot\alpha}{\sin\alpha} \qquad 2\sin\alpha < \sin 2\alpha$$

$$\frac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha} > 0 \rightarrow \frac{\cos\alpha}{\sin^2\alpha} > 0 \rightarrow \cos\alpha > 0$$

$$\cancel{2}\sin\alpha < \cancel{2}\sin\alpha \underbrace{\cos\alpha}_{+} \Rightarrow \sin\alpha < 0$$

$0 < \sin < 1$

$\cos\alpha +$
$\sin\alpha -$
$\boxed{\text{ناحیه چهارم}}$