تکلیف شماره 28 [illegible] یازدهم دختر

1)

$$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \Rightarrow \frac{1}{|\cos\alpha|} - \tan\alpha = \frac{1-\sin\alpha}{|\cos\alpha|}$$

$$\frac{\cancel{1}-\cancel{1}+\sin\alpha}{|\cos\alpha|} = \frac{\sin\alpha}{\cos\alpha} \Rightarrow |\cos\alpha| = \cos\alpha$$

cos و sin مثبت هستند ← ناحیه اول

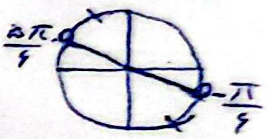
$$\cot\alpha = \frac{\cos\alpha}{\sqrt{\underbrace{1-\cos^2\alpha}_{\sin^2\alpha}}} \Rightarrow \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow |\sin\alpha| = \sin\alpha$$

---

2)

$$\frac{-\pi}{12} < x < \frac{5\pi}{12} \to \frac{-\pi}{6} < 2x < \frac{5\pi}{6} \to \frac{-1}{2} < \sin 2x \le 1$$

$$\frac{-1}{2} < \frac{m-1}{4} \le 1 \to -2 < m-1 \le 4 \to -1 < m \le 5$$



مجموعه مقادیر m $\Rightarrow (-1, 5]$

---

3)

$$\tan x + \cot x = -3 \to \frac{\overbrace{\sin^2 x+\cos^2 x}^{1}}{\sin x\cos x} = -3 \to \sin x\cos x = \frac{-1}{3}$$

$$\sin^3 x+\cos^3 x = (\sin x+\cos x)(\underbrace{\sin^2 x+\cos^2 x}_{1} - \underbrace{\sin x\cos x}_{\frac{-1}{3}}) = \frac{-1}{\sqrt{3}} \times \frac{4}{3} = \frac{-4}{3\sqrt{3}}$$

$$(\sin x+\cos x)^2 = \sin^2 x+\cos^2 x+\sin x\cos x = 1-\frac{2}{3} = \frac{1}{3} \to \sin x+\cos x = \pm\frac{1}{\sqrt{3}}$$

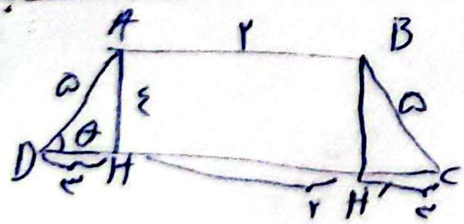
غ ق ق

$$3\pi < 4x < 4\pi \to \frac{3\pi}{4} < x < \pi \to -1 < \cos x < \frac{-\sqrt{2}}{2},\ 0 < \sin x < \frac{\sqrt{2}}{2}$$

$$\to \sin x+\cos x < 0 \qquad \frac{1}{\sin^3 x+\cos^3 x} = \frac{1}{\frac{-4}{3\sqrt{3}}} = \frac{-3\sqrt{3}}{4}$$

---

4)



$$\cos\theta = \frac{DH}{AD} = \frac{DH}{5} = \frac{6}{10} \Rightarrow DH = 3$$

$$\to CH' = 3 \qquad AB = HH' = 2$$

$$\to CD = 3+3+2 = 8 \qquad AD^2 = DH^2 + AH^2$$

$$5^2 = 3^2 + AH^2 \qquad AH = 4$$

$$S_{\text{ذوزنقه}} = \frac{(AB+CD)\times AH}{2} = \frac{(2+8)\times 4}{2} = 20$$

۵)

$$\tan(۲۸۵^\circ)\tan(-۱۶۵) - \sin(۱۰۹۵^\circ)\cos(۲۵۵^\circ)$$

$$= \tan\left(\frac{۳\pi}{۲}+۱۵\right)\times\tan(-\pi+۱۵) - \sin(۶\pi+۱۵)\times\cos\left(\frac{۳\pi}{۲}-۱۵\right)$$

$$= -\cot ۱۵ \times \tan ۱۵ - \sin ۱۵ \times -\sin ۱۵ = -۱ + \sin^۲ ۱۵$$

$$= -۱ + (۱-\cos^۲ ۱۵) = -\cos^۲ ۱۵ = k\cos^۲ ۱۵ \Rightarrow \boxed{k=-۱}$$

---

۶)

$$A = \sqrt{۳}\cos(۲۱۰)\times\sin(۲۴۳) - \sqrt{۲}\sin(۱۳۵)\cos(۱۵۳)$$

$$= \sqrt{۳}\times\frac{-\sqrt{۳}}{۲}\times\sin\left(\frac{۳\pi}{۲}-۲۷\right) - \sqrt{۲}\times\frac{\sqrt{۲}}{۲}\times\cos(\pi-۲۷)$$

$$= \frac{-۳}{۲}\times-\cos ۲۷ - ۱\times-\cos ۲۷ = \frac{۳}{۲}\cos ۲۷ + \cos ۲۷ = \frac{۵}{۲}\cos ۲۷$$

$$\frac{A}{\cos ۲۷^\circ} = \frac{\frac{۵}{۲}\cos ۲۷^\circ}{\cos ۲۷^\circ} = \frac{۵}{۲}$$

---

۷)

$$f\left(\frac{\pi}{۲۴}\right) = ۱۶\cos^۲\left(\frac{\pi}{۱۲}\right)\times\underbrace{\cos^۲\left(\frac{\pi}{۶}\right)}_{\frac{۱}{۴}}\times\underbrace{\cos^۲\left(\frac{\pi}{۳}\right)}_{\frac{۳}{۴}}\times\underbrace{\cos^۲\left(\frac{۲\pi}{۳}\right)}_{\frac{۱}{۴}}$$

$$= \cancel{۱۶}\times\frac{۲+\sqrt{۳}}{۴}\times\frac{۱}{\cancel{۴}}\times\frac{۳}{۴}\times\frac{۱}{\cancel{۴}} = \frac{۶+۳\sqrt{۳}}{۱۶}$$

$$\cos^۲\left(\frac{\pi}{۱۲}\right) = \frac{۱+\cos\frac{\pi}{۶}}{۲} = \frac{۱+\frac{\sqrt{۳}}{۲}}{۲} = \frac{\frac{۲+\sqrt{۳}}{۲}}{۲} = \frac{۲+\sqrt{۳}}{۴}$$

---

۸)

$$\frac{۱-\sin x}{۱+\sin x} = ۴ \rightarrow ۱-\sin x = ۴+۴\sin x \rightarrow ۵\sin x = -۳$$

$$\sin x = \frac{-۳}{۵}$$

$$\sin^۲ x + \cos^۲ x = ۱ \rightarrow \frac{۹}{۲۵} + \cos^۲ x = ۱ \rightarrow \cos x = \frac{-۴}{۵} \checkmark$$

$$\cos x = \frac{۴}{۵}$$ غ ق

$$\tan\frac{x}{۲} = \frac{\sin x}{۱+\cos x} = \frac{\frac{-۳}{۵}}{۱+\frac{-۴}{۵}} = \frac{\frac{-۳}{۵}}{\frac{۱}{۵}} = -۳$$

۹)

$$\frac{\sin\theta}{1-\cos\theta}+\frac{1+\cos\theta}{\sin\theta}\xrightarrow{\text{با توجه به اینکه}}\quad \frac{\sin\theta}{1+\cos\theta}=\frac{1-\cos\theta}{\sin\theta}=\tan\frac{\theta}{2}$$

$$=\cot\frac{\theta}{2}+\cot\frac{\theta}{2}=2\cot\frac{\theta}{2}=k\cot\frac{\theta}{2}\Longrightarrow\boxed{k=2}$$

---

۱۰)

$$\cos\left(\frac{11\pi}{4}+\alpha\right)=\underbrace{\cos\frac{11\pi}{4}}_{-\frac{\sqrt{2}}{2}}\cos\alpha-\underbrace{\sin\frac{11\pi}{4}}_{\frac{\sqrt{2}}{2}}\sin\alpha$$

$$=\frac{-\sqrt{2}}{2}\times\frac{-7\sqrt{2}}{10}-\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{10}=\frac{7}{10}-\frac{1}{10}=\frac{6}{10}=\frac{3}{5}$$

$$\sin^2\alpha+\cos^2\alpha=1\longrightarrow\frac{2}{100}+\cos^2\alpha=1$$

$\cos\alpha=+\frac{7\sqrt{2}}{10}$ غقق

$\cos\alpha=\frac{-7\sqrt{2}}{10}$ ✓
