تکلیف ۳۵ / حسنا ضیائی / دختر یازدهم C

(۲۰)

1

$$\lim_{x\to 1}\frac{4x^2-7x+3}{5x^2-8x+3}=\frac{0}{0}\xrightarrow[\text{ابهام}]{\text{رفع}}\frac{(x-1)(4x-3)}{(x-1)(5x-3)}=\frac{1}{2}$$

(۵)

2

$$\lim_{x\to 0}\frac{|3x-1|-|3x+1|}{x}=\frac{0}{0}\xrightarrow[\text{ابهام}]{\text{رفع}}\frac{1-3x-3x-1}{x}=\frac{-6x}{x}=-6$$

(۲)

3

$$\lim_{x\to 4}\frac{x-4}{\sqrt{x}-2}=\frac{0}{0}\xrightarrow[\text{ابهام}]{\text{رفع}}\frac{(\sqrt{x}-2)(\sqrt{x}+2)}{\sqrt{x}-2}=\sqrt{4}+2=4$$

(۵)

4

$$\lim_{x\to 2}\frac{x-\sqrt{3x}}{3x^2-x-4}=\frac{0}{0}\xrightarrow[\text{ابهام}]{\text{رفع}}\frac{x-\sqrt{3x}}{3x^2-x-4}\times\frac{x+\sqrt{3x}}{x+\sqrt{3x}}=\frac{x^2-3x}{(x-2)(3x+4)}\times\frac{1}{x+\sqrt{3x}}$$

$3x^2-x-4$ با روش روشی $\to 12-x-3x^2$ $\to (3x+4)(4-x)$ $\to x=\frac{3}{2}\pm\frac{5}{2}\to 4,-1$ ریشه‌های مخرج

$$\frac{x(x-3)}{(x-2)(3x+4)(x+\sqrt{3x})}\to\frac{2}{7}\times\frac{1}{4}=\frac{1}{14}$$

(۵)

5

$$\lim_{x\to 1}\frac{1-\sqrt{x}}{2-\sqrt{5-x}}=\frac{0}{0}\xrightarrow[\text{ابهام}]{\text{رفع}}\frac{1-x}{4-(5-x)}\times\frac{2+\sqrt{5-x}}{2-\sqrt{5-x}}$$

$$\to\frac{1-x}{x-1}\times\frac{2+\sqrt{5-x}}{2-\sqrt{5-x}}=-(2+\sqrt{5-x})(2+\sqrt{5-x})$$

$$\to -4\times\frac{-1}{4}=-4$$

Parsian

6

$$\lim_{x\to 4} \frac{\sqrt{3x+4}-\sqrt{9x-4}}{\sqrt{5x+5}-3} = \frac{0}{0} \Rightarrow \text{ضرب مزدوج}$$

$$\frac{3x+4-9x+4}{-6x+8} \times \frac{\sqrt{5x+5}+3}{1} \times \frac{\sqrt{3x+4}+\sqrt{9x-4}}{1}$$

$$\frac{-6x+8}{-6x+8} \times \frac{\sqrt{5x+5}+3}{1} \times \frac{\sqrt{3x+4}+\sqrt{9x-4}}{1}$$

$$= 1 \times \frac{3+3}{1} \times \frac{\sqrt{16}+\sqrt{32}}{1} = 6 \times (4+4\sqrt{2}) = 24+24\sqrt{2}$$

7

$$\lim_{x\to 1} \frac{\sqrt{x^3+\sqrt{x}}-2}{\sqrt[3]{x}-1} = \frac{0}{0} \Rightarrow \text{ضرب مزدوج}$$

$$\frac{x^3+\sqrt{x}-4}{\sqrt[3]{x}-1} \times \frac{\sqrt{x^3+\sqrt{x}}+2}{\sqrt{x^3+\sqrt{x}}+2} \times \frac{\sqrt[3]{x^2}+\sqrt[3]{x}+1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1}$$

$$\frac{x^3-1+\sqrt{x}-1}{x-1} \times \frac{\sqrt[3]{x^2}+\sqrt[3]{x}+1}{\sqrt{x^3+\sqrt{x}}+2} = \left(\frac{(x-1)(x^2+x+1)}{x-1} + \frac{\sqrt{x}-1}{x-1}\right) \times \frac{3}{4}$$

$$= \left((x^2+x+1) + \frac{\sqrt{x}-1}{(\sqrt{x}-1)(\sqrt{x}+1)}\right) \times \frac{3}{4} = \left(3 + \frac{1}{\sqrt{x}+1}\right) \times \frac{3}{4} = \frac{3}{4} + \frac{3}{8} = \frac{9}{8}$$

8

$$\lim_{x\to \pi} \frac{1+\cos^3 x}{\sin^2 x} = \frac{0}{0} \Rightarrow \text{اتحاد}$$

$$\frac{(1+\cos x)(1-\cos x+\cos^2 x)}{1-\cos^2 x} = \frac{1-\cos x+\cos^2 x}{1-\cos x} = \frac{\frac{1}{2}+\frac{3}{4}}{1+\frac{1}{2}} = \frac{\frac{3}{4}}{\frac{3}{2}} = \frac{1}{2}$$

9

$$\lim_{x\to \frac{\pi}{4}} \frac{1-\tan x}{\sin x-\cos x} = \frac{0}{0} \Rightarrow \text{اتحاد}$$

$$\frac{1-\frac{\sin x}{\cos x}}{\sin x-\cos x} = \frac{\frac{\cos x-\sin x}{\cos x}}{\sin x-\cos x} = \frac{-1}{\cos x} = \frac{-1}{\frac{\sqrt{2}}{2}} = -\sqrt{2}$$

10

$$\lim_{x\to \frac{\pi}{4}} \frac{\tan^2 x-1}{\cos 2x} = \frac{0}{0} \Rightarrow \text{اتحاد}$$

$$\frac{\frac{\sin^2 x}{\cos^2 x}-1}{\cos^2 x-\sin^2 x} = \frac{\frac{\sin^2 x-\cos^2 x}{\cos^2 x}}{\cos^2 x-\sin^2 x} = \frac{-1}{\cos^2 x} = \frac{-1}{\frac{1}{2}} = -2$$