۱) $\lim_{x\to 1} \frac{2x^2-7x+3}{5x^2-8x+3} \to \frac{2-7+3}{5-8+3} = \frac{0}{0} \xrightarrow{\text{hop}} \frac{4x-7}{10x-8} \to \frac{4-7}{10-8} = \boxed{\frac{1}{2}}$

۲) $\star \lim_{x\to 0} \frac{|2x-1|-|2x+1|}{x} \to \frac{1+2x+1+2x-1}{x} = \frac{0}{0} \to \frac{4x}{x} = 4 = \boxed{-4}$

۳) $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} \to \frac{0}{0} \xrightarrow{\text{hop}} \frac{1}{\frac{1}{2\sqrt{x}}} = 2\sqrt{x} \to \boxed{4}$

۴) $\star\star \lim_{x\to 2} \frac{x-\sqrt{2x}}{2x^2-x-6} \to \frac{0}{0} \xrightarrow{\text{hop}} \frac{1-\frac{2}{2\sqrt{2x}}}{4x-1} = \frac{2\sqrt{2x}-2}{(4x-1)2\sqrt{2x}} = \frac{2}{28} = \boxed{\frac{1}{14}}$

۵) $\lim_{x\to 1} \frac{1-\sqrt{x}}{2-\sqrt{5-x}} \to \frac{0}{0} \xrightarrow{\text{hop}} \frac{-\frac{1}{2\sqrt{x}}}{\frac{-1}{2\sqrt{5-x}}} = \frac{2\sqrt{5-x}}{-2\sqrt{x}} \to \frac{4}{-2} = \boxed{-2}$

(٦)

$$\lim_{x\to 2} \frac{\sqrt{3x+3}-3}{\sqrt{5x+1}-3} \to \frac{0}{0} \xrightarrow{\text{hop}} \frac{\frac{3}{2\sqrt{3x+3}}}{\frac{5}{2\sqrt{(5x+1)^3}}} = \frac{17}{40}$$

$$x^{\frac{1}{2}} \to \frac{1}{2}x^{-\frac{1}{2}} \quad \frac{1}{2} \quad \frac{1}{2\sqrt{x}}$$

(٧)

$$\lim_{x\to 1} \frac{\sqrt{3x+\sqrt{x}}-2}{\sqrt{x}-1} \to \frac{0}{0} \xrightarrow{\text{hop}} \frac{1\times\left(3+\frac{1}{2\sqrt{x}}\right)}{2\sqrt{3x+\sqrt{x}}} = \frac{\frac{7}{2}}{\frac{4}{1}} = \frac{7}{8}$$

$3+\frac{1}{2} \to \frac{7}{2}$

(٨)

$$\lim_{x\to\pi} \frac{1+\cos^2 x}{\sin^2 x} \to \frac{2}{0} \to \frac{(1+\cos x)(1+\cos^2 x-\cos x)}{1-\cos^2 x} = \frac{3}{2}$$

$1+1+1$

$(1+\cos x)(1-\cos x)$

$1+1$

(٩)

$$\lim_{x\to\frac{\pi}{4}} \frac{1-\tan\alpha}{\sin\alpha-\cos\alpha} \to \frac{\cos\alpha}{\cos\alpha}-\frac{\sin\alpha}{\cos\alpha} \to \frac{-(\sin-\cos)}{\cos} = \frac{-1}{\cos} = \frac{-1}{\frac{\sqrt{2}}{2}} = \frac{-2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = -\sqrt{2}$$

$$\frac{\frac{\sin-\cos}{1}}{\frac{\cos}{1}} = \frac{\cos-\sin}{\cos}$$

(١٠)

$$\lim_{x\to\frac{3\pi}{4}} \frac{\tan^2 x-1}{\cos 2x} \to \frac{(\tan x+1)(\tan x-1)}{\cos^2 x-\sin^2 x}$$

$\tan\frac{3\pi}{4} = -1$

$$\frac{\frac{\tan^2\alpha-1}{1}}{\frac{-(\tan^2\alpha-1)}{1+\tan^2\alpha}} = \frac{1+\tan^2\alpha}{-1} = -2$$