تمرینات تکمیلی — تألیف نگارنده — ۱۸۵

۱) $\lim_{x\to 1} \frac{2x^2-5x+3}{3x^2-4x+1} = \frac{0}{0}$ مبهم $\to \lim_{x\to 1} \frac{2(x-1)(x-\frac{3}{2})}{3(x-1)(x-\frac{1}{3})} = \frac{2\times\frac{1}{2}}{3\times\frac{2}{3}} = \frac{1}{2}$

۲) $\lim_{x\to 0} \frac{|2x-1|-|2x+1|}{x} = \frac{0}{0}$ مبهم $\to \lim_{x\to 0} \frac{-2x+1-2x-1}{x} = \lim_{x\to 0} \frac{-4x}{x} = -4$

۳) $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = \frac{0}{0} \to \lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = \lim_{x\to 4} \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{\sqrt{x}-2} = 4$

۴) $\lim_{x\to 2} \frac{x-\sqrt{2x}}{2x^2-x-6} = \frac{0}{0} \to \lim_{x\to 2} \frac{x-\sqrt{2x}}{2x^2-x-6} \times \frac{x+\sqrt{2x}}{x+\sqrt{2x}} = \lim_{x\to 2} \frac{x^2-2x}{2x^2-x-6} \times \frac{1}{x+\sqrt{2x}}$

$x^2-x-12 = (x+3)(x-4)$

$= \lim_{x\to 2} \frac{x(x-2)}{(x-2)(x+\frac{3}{2})} \times \frac{1}{x+\sqrt{2x}} = \frac{2}{2\times\frac{7}{2}\times 2} = \frac{1}{10}$

۵) $\lim_{x\to 1} \frac{1-\sqrt{x}}{2-\sqrt{5-x}} = \frac{0}{0} \to \lim_{x\to 1} \frac{1-\sqrt{x}}{2-\sqrt{5-x}} \times \frac{1+\sqrt{x}}{1+\sqrt{x}} \times \frac{2+\sqrt{5-x}}{2+\sqrt{5-x}} = \lim_{x\to 1} \frac{1-x}{-(x-1)} \times \frac{(2+\sqrt{5-x})}{1+\sqrt{x}} = -1 \times \frac{4}{2} = -2$

۶) $\lim_{x\to 3} \frac{\sqrt{3x+7}-5}{\sqrt[3]{2x+1}-3} = \frac{0}{0} \to \lim_{x\to 3} \frac{\sqrt{3x+7}-5}{\sqrt[3]{2x+1}-3} \times \frac{\sqrt{3x+7}+5}{\sqrt{3x+7}+5} \times \frac{\sqrt[3]{(2x+1)^2}+9+3\sqrt[3]{2x+1}}{\sqrt[3]{(2x+1)^2}+9+3\sqrt[3]{2x+1}}$

$= \lim_{x\to 3} \frac{3x-12}{2x-7} \times \frac{\sqrt[3]{(2x+1)^2}+9+3\sqrt[3]{2x+1}}{\sqrt{3x+7}+5} = \frac{3}{2} \times \frac{27}{10} = \frac{81}{20}$

۷) $\lim_{x\to 1} \frac{\sqrt{3x+\sqrt{x}}-2}{\sqrt[3]{x}-1} = \frac{0}{0} \to \lim_{x\to 1} \frac{\sqrt{3x+\sqrt{x}}-2}{\sqrt[3]{x}-1} \times \frac{\sqrt{3x+\sqrt{x}}+2}{\sqrt{3x+\sqrt{x}}+2} \times \frac{\sqrt[3]{x^2}+1+\sqrt[3]{x}}{\sqrt[3]{x^2}+1+\sqrt[3]{x}}$

$= \lim_{x\to 1} \frac{3x+\sqrt{x}-4}{x-1} \times \frac{3}{4} \times \frac{((3x-2)+\sqrt{x})}{((3x-2)+\sqrt{x})} = \lim_{x\to 1} \frac{9x^2-5x+14}{x-1} \times \frac{3}{4(-2)} = \lim_{x\to 1} \frac{(x-1)(9x-14)}{(x-1)} \times \frac{3}{-8}$

$= (-7)\left(\frac{3}{-8}\right) = \frac{21}{8}$

۸) $\lim_{x\to \pi} \frac{1+\cos^3 x}{\sin^2 x} = \frac{0}{0} \to \lim_{x\to \pi} \frac{(1+\cos x)(1+\cos^2 x-\cos x)}{(1-\cos x)(1+\cos x)} = \frac{1+1-(-1)}{1-(-1)^2} = \frac{3}{2}$

$1-\cos^2 x$

۹) $\lim_{x\to \frac{\pi}{4}} \frac{1-\tan x}{\sin x-\cos x} = \frac{0}{0} \to \lim_{x\to \frac{\pi}{4}} \frac{1-\frac{\sin x}{\cos x}}{\sin x-\cos x} = \lim_{x\to \frac{\pi}{4}} \frac{\frac{\cos x-\sin x}{\cos x}}{\sin x-\cos x} = \frac{-1}{\frac{\sqrt{2}}{2}} = -\sqrt{2}$

۱۰) $\lim_{x\to \frac{\pi}{4}} \frac{\tan^2 x-1}{\cos 2x} = \frac{0}{0} \to \lim_{x\to \frac{\pi}{4}} \frac{\frac{\sin^2 x}{\cos^2 x}-1}{\cos^2 x-\sin^2 x} = \frac{\frac{\sin^2 x-\cos^2 x}{\cos^2 x}}{\cos^2 x-\sin^2 x} = \frac{-1}{\frac{1}{2}} = -2$

۶) حاصل $\to \frac{1-\frac{2}{2\sqrt{2x}}}{4x-1} = \frac{1}{14}$