حسابان تشریحی — تکلیف شماره ۳ — یازدهم ریاضی

۱) $\lim_{x\to 1} \frac{4x^2-7x+3}{5x^2-8x+3} = \frac{0}{0}$ ابهام $\longrightarrow \lim_{x\to 1} \frac{4\cancel{(x-1)}(x-\frac{3}{4})}{5\cancel{(x-1)}(x-\frac{3}{5})} = \frac{4\times\frac{1}{4}}{5\times\frac{2}{5}} = \frac{1}{2}$

۲) $\lim_{x\to 0} \frac{|3x-1|-|3x+1|}{x} = \frac{0}{0}$ ابهام $\longrightarrow \lim_{x\to 0} \frac{-3x+1-3x-1}{x} = \lim_{x\to 0} \frac{-6x}{x} = -6$

۳) $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = \frac{0}{0} \longrightarrow \lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = \lim_{x\to 4} \frac{\cancel{(\sqrt{x}-2)}(\sqrt{x}+2)}{\cancel{\sqrt{x}-2}} = 4$

۴) $\lim_{x\to 2} \frac{x-\sqrt{2x}}{2x^2-x-6} = \frac{0}{0} \longrightarrow \lim_{x\to 2} \frac{x-\sqrt{2x}}{2x^2-x-6} \times \frac{x+\sqrt{2x}}{x+\sqrt{2x}} = \lim_{x\to 2} \frac{x^2-2x}{2x^2-x-6} \cdot \frac{1}{x+\sqrt{2x}} =$

$x^2-\frac{x}{2}-3$, $(x+\frac{3}{2})(x-\frac{4}{2})$

$\lim_{x\to 2} \frac{x\cancel{(x-2)}}{2\cancel{(x-2)}(x+\frac{3}{2})} \times \frac{1}{x+\sqrt{2x}} = \frac{\cancel{2}}{\cancel{2}\times\frac{7}{2}\times 4} = \frac{1}{10}$

۵) $\lim_{x\to 1} \frac{1-\sqrt{x}}{2-\sqrt{5-x}} = \frac{0}{0} \longrightarrow \lim_{x\to 1} \frac{1-\sqrt{x}}{2-\sqrt{5-x}} \times \frac{1+\sqrt{x}}{1+\sqrt{x}} \times \frac{2+\sqrt{5-x}}{2+\sqrt{5-x}} = \lim_{x\to 1} \frac{\cancel{1-x}}{\cancel{x-1}} \times \frac{(2+\sqrt{5-x})}{1+\sqrt{x}} = -1 \times \frac{4}{2} = -2$

۶) $\lim_{x\to 4} \frac{\sqrt{3x+4}-4}{\sqrt[3]{5x+7}-3} = \frac{0}{0} \longrightarrow \lim_{x\to 4} \frac{\sqrt{3x+4}-4}{\sqrt[3]{5x+7}-3} \times \frac{\sqrt{3x+4}+4}{\sqrt{3x+4}+4} \times \frac{\sqrt[3]{(5x+7)^2}+9+3\sqrt[3]{5x+7}}{\sqrt[3]{(5x+7)^2}+9+3\sqrt[3]{5x+7}}$

$= \lim_{x\to 4} \frac{\overset{3(x-4)}{3x-12}}{\underset{5(x-4)}{5x-20}} \times \frac{\sqrt[3]{(5x+7)^2}+9+3\sqrt[3]{5x+7}}{\sqrt{3x+4}+4} = \frac{3}{5}\times\frac{27}{8} = \frac{81}{40}$

۷) $\lim_{x\to 1} \frac{\sqrt{3x+\sqrt{x}}-2}{\sqrt[3]{x}-1} = \frac{0}{0} \longrightarrow \lim_{x\to 1} \frac{\sqrt{3x+\sqrt{x}}-2}{\sqrt[3]{x}-1} \times \frac{\sqrt{3x+\sqrt{x}}+2}{\sqrt{3x+\sqrt{x}}+2} \times \frac{\sqrt[3]{x^2}+1+\sqrt[3]{x}}{\sqrt[3]{x^2}+1+\sqrt[3]{x}}$

$= \lim_{x\to 1} \frac{3x+\sqrt{x}-4}{x-1} \times \frac{3}{4} \times \frac{((3x-4)-\sqrt{x})}{((3x-4)-\sqrt{x})} = \lim_{x\to 1} \frac{9x^2-25x+16}{x-1} \times \frac{3}{4\times(-2)} = \lim \frac{\cancel{(x-1)}(9x-16)}{\cancel{(x-1)}} \times \frac{3}{-8}$

$= (-7)\left(\frac{3}{-8}\right) = \frac{21}{8}$

۸) $\lim_{x\to \pi} \frac{1+\cos^3 x}{\sin^2 x} = \frac{0}{0} \longrightarrow \lim_{x\to \pi} \frac{\cancel{(1+\cos x)}(1+\cos^2 x-\cos x)}{\underset{1-\cos^2 x}{(1-\cos x)\cancel{(1+\cos x)}}} = \frac{1+1-(-1)}{1-(-1)} = \frac{3}{2}$

۹) $\lim_{x\to \frac{\pi}{4}} \frac{1-\tan x}{\sin x-\cos x} = \frac{0}{0} \longrightarrow \lim_{x\to \frac{\pi}{4}} \frac{1-\frac{\sin x}{\cos x}}{\sin x-\cos x} = \lim_{x\to \frac{\pi}{4}} \frac{\frac{\cancel{\cos x-\sin x}}{\cos x}}{\cancel{\sin x-\cos x}} = \frac{-1}{\frac{\sqrt{2}}{2}} = \frac{-2}{\sqrt{2}}$

۱۰) $\lim_{x\to \frac{3\pi}{4}} \frac{\tan^2 x-1}{\cos 2x} = \frac{0}{0} \longrightarrow \lim_{x\to \frac{3\pi}{4}} \frac{\frac{\sin^2 x}{\cos^2 x}-1}{\cos^2 x-\sin^2 x} = \frac{\frac{\cancel{\sin^2 x-\cos^2 x}}{\cos^2 x}}{\cancel{\cos^2 x-\sin^2 x}} = \frac{-1}{\frac{1}{2}} = -2$